Basics of Physical Acoustics

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Contents

0. Introduction

This part of tutorial deals with basic concepts related to generation of sound, propagation of sound through media, and sound sources. Topics are divided in three parts what includes (1) physical properties of simple vibration systems which provide the basics for vibration and oscillatory motion of particles in acoustical wave in the medium ; (2) the physical properties of sound waves, their mathematical description, speed of sound and phenomena related with sound propagation in unbounded space and space with boundaries of other media or obstacles causing reflection, rarefaction, and diffraction of sound waves; and (3) elementary models of sound sources.

1. Simple Harmonic Motion

1.1 Lossless oscillator

Simple harmonic motion is a periodic motion along a straight line by a mass with acceleration proportional to the distance from the center point. For such a linear system, the position of the mass is described by a sinusoidal function of time t with the amplitude A and frequency f (period $T = 1/f$): $x(t) = A \sin 2\pi ft$.

Harmonic motion is typical for the mass-spring system shown in Fig. 1.1, in which restoring force F imposed by a mechanical spring is proportional to stretch x. There are other relations between F and x considered in mechanics but for acoustics linear systems well represent majority of phenomena. In a lossless system, it is assumed that there is no loss of energy in such a system. The equation of motion is a combination of Hooke's law, $F = -Kx$, and Newton's second law, $F = ma = m\ddot{x}$:

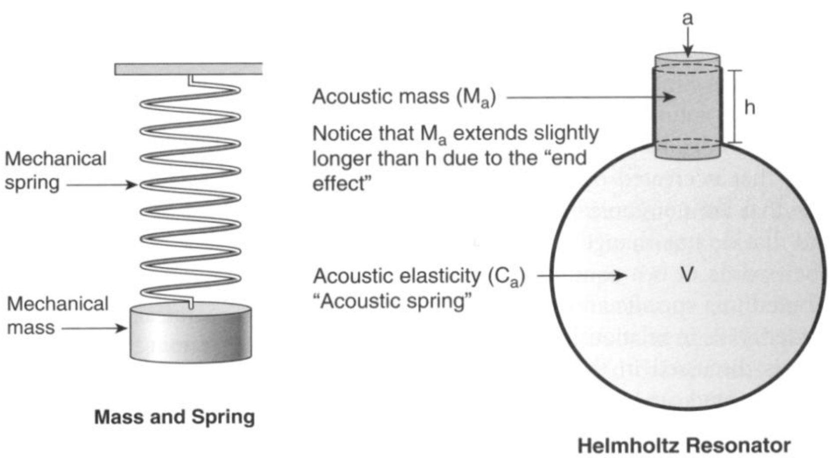


Fig. 1.1. Simple mass-spring vibrating system and its acoustic equivalent, the Helmholtz resonator. From reference [1].

The constant K represents stiffness of the spring in Newtons per meter (N/m). The equation for forces in the system is:

$$m\ddot{x} + Kx = 0, \tag{1.1}$$

where

$$\ddot{x} = \frac{d^2x}{dt^2} \tag{1.2}$$

The equation solves to the momentary position:

$$x = A\cos(\omega_0 t + \phi) \tag{1.3}$$

where
$$\omega_0 = \sqrt{K/m} \tag{1.4}$$

is the natural angular frequency of the system whose natural frequency f_0 fulfils relation $\omega_0 = 2\pi f_0$.

By differentiating and double differentiating, the velocity and acceleration in the system can be calculated whose amplitudes are multiplication by ω_0 and ω_0^2, respectively and phases are such that the velocity is shifted by 90^o (maximum velocity is while passing the $x = 0$ center point), and acceleration is opposite in phase (180^o) to the position (for maximum shift away from the center the acceleration is the largest towards the center).

The amplitude and initial phase of motion results from a combination of the initial displacement x_0 and initial velocity v_0 applied to the system:

$$A = \sqrt{x_0^2 + \left(\frac{v_0}{\omega_0}\right)^2}, \qquad \phi = \tan^{-1}\left(\frac{-v_0}{\omega_0 x_0}\right). \tag{1.5}$$

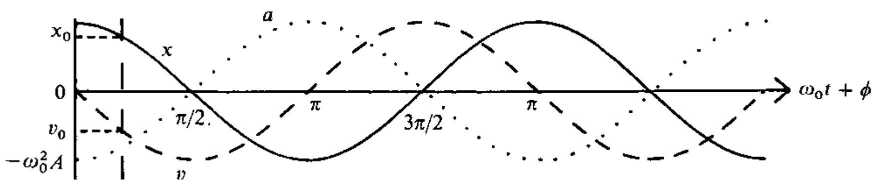


Fig. 1.2. Relative phase of displacement x, velocity v, and acceleration $\ddot{x}$ of a simple vibrating mechanical system. From reference [2].

The energy of harmonic motion is $E = KA^2/2 = mv_{max}^2$, where K is the stiffness constant, and v_{max} the maximum velocity. For more details see [2].

1.2. Superposition of Harmonic Motions

Representing trigonometric functions with complex exponential functions in symbolic notation, such as

$$\begin{aligned} x &= A\cos(\omega_0 t + \phi) = \mathrm{Re}[Ae^{j(\omega_0 t + \phi)}] = \mathrm{Re}(Ae^{j\phi}e^{j\omega_0 t}) \\ &= \mathrm{Re}(\tilde{A}e^{j\omega_0 t}). \end{aligned} \tag{1.6}$$

allows to consider superposition of harmonic motions as sum of vectors (phasors) rotating in the complex plane with angular velocity ω_0. The real time dependence of each quantity can be obtained from the projection on the real axis.

Such a superposition is shown in Fig. 1.3 for phasors representing harmonic motions of amplitudes A_1 and A_2, and phases Φ_1 and Φ_2. Resulting signal has an amplitude A and phase Φ.

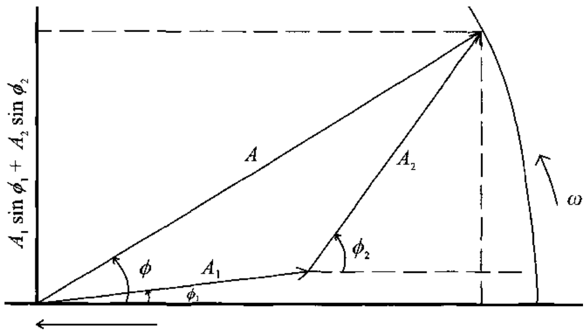


Fig. 1.3. Phasor representation of two simple harmonic motions having the same frequency. From reference [2].

1.2.1. n Harmonic motions of the same frequency

There are two phasors added up in Fig. 1.3. In more general case *n* phasors can be added. This results in an amplitude *A* and phase *Φ* of a sum of *n* harmonic motions as

$$A = \sqrt{(\sum A_n \cos\phi_n)^2 + (\sum A_n \sin\phi_n)^2}, \tag{1.7}$$

and

$$\tan\phi = \frac{\sum A_n \sin\phi_n}{\sum A_n \cos\phi_n} \quad . \tag{1.8}$$

1.2.2. Beats: two harmonic motions with different frequencies:

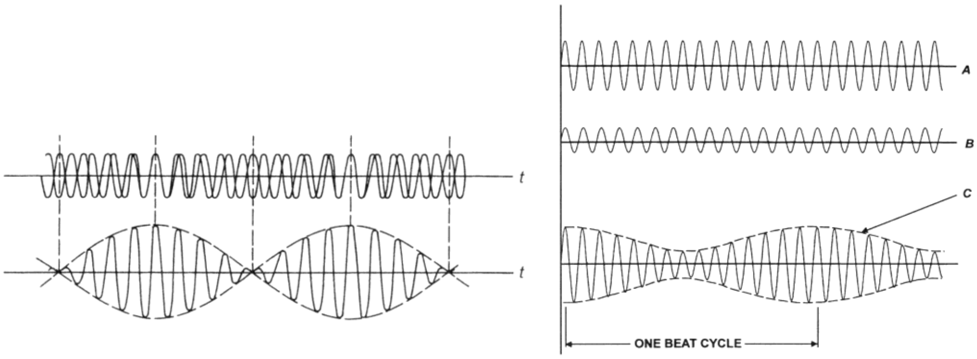
A combination of harmonic motions of frequency f_1 and f_2, for small $\Delta f = f_2 - f_1$ leads to periodic amplitude variations that create a sensation of beats. For

$$\tilde{x} = \tilde{x}_1 + \tilde{x}_2 = A_1 e^{j(\omega_1 t + \phi_1)} + A_2 e^{j(\omega_2 t + \phi_2)}, \tag{1.9}$$

when $A_1 = A_2$ then (phases are disregarded)

$$x(t) = 2A\cos(2\pi\,(\Delta f/2)\,t)\cos(2\pi\,((f_1+f_2)/2)\,t) \tag{1.10}$$

the amplitude varies for zero to 2*A*. Corresponding beats are very pronounced (Fig. 1.4a). When amplitudes A_1 and A_2 are different the amplitude of their sum does not decrease to zero (Fig. 1.4b). Respective formulas are more complicated, or details see for example [2].



a) b)

Fig. 1.4. Waveform resulting from linear superposition producing beats a) same amplitudes. b) different amplitudes. From references [2] and [5].

Formula (10) represents amplitude modulation with carrier absent> Please note, that while the modulation frequency is $\Delta f/2 = (f_2 - f_2)/2$ the frequency of beats (loudness change) is Δf.

1.3. Damped oscillations

Damping of system with harmonic motion is caused by loss of energy coming at most from viscosity of the fluid. In this case, the damping force is proportional to velocity

$$F = -R\ddot{x}, \tag{1.11}$$

where R is the mechanical resistance. The equation of motion for damped system is:

$$m\ddot{x} + R\dot{x} + Kx = 0 \tag{1.12}$$

or

$$\ddot{x} + 2\alpha\dot{x} + \omega_0^2 x = 0, \tag{1.13}$$

where $\alpha = R/2m$ and $\omega_0^2 = K/m.$

The solution of this equation represents cosine-like variation of angular frequency ω_d weighted exponentially by $e^{-\alpha t}$

$$x = Ae^{-\alpha t}\cos(\omega_d t + \phi), \tag{1.14}$$

where

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2}\;. \tag{1.15}$$

The damped oscillations for various α values are shown in Figs. 3 and 8

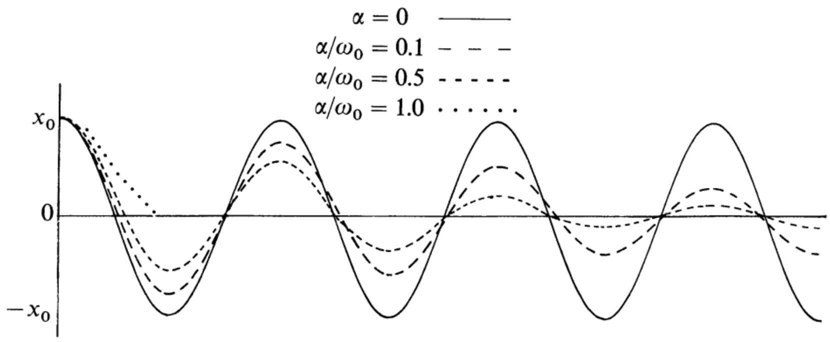


Fig. 1.5. Displacement of a harmonic oscillator for different values of damping. The relaxation time is given by $1/\alpha$. Critical damping occurs when $\alpha = \omega_0$. From reference [2].

Basic parameters of damped oscillations include:

a) relaxation time, which is a time after which the amplitude decreases *e* times

$$\tau = \frac{1}{\alpha} = \frac{2m}{R} \tag{1.16}$$

b) the rate of energy loss

$$d/dt(E_p + E_K) = \dot{x}(-R\dot{x}) = -2\alpha m\dot{x}^2, \tag{1.17}$$

c) the Q (Quality) factor, which is a ratio of spring force to the damping force

$$Q = \frac{Kx_0}{R\omega_0 x_0} = \frac{K}{R\omega_0} = \frac{\omega_0}{2\alpha}. \tag{1.18}$$

1.4. Forced oscillations

Forced oscillations occur when an oscillator is driven by an external force $f(t)$ (see Fig. 1.6). Analysis of such a system is commonly done for a sinusoidal driving force $f(t) = F \cos \omega t$ turned on at some time. The solution consists of a transient term, and a steady-state term that depends only on F and ω

$$m\ddot{\tilde{x}} + R\dot{\tilde{x}} + K\tilde{x} = Fe^{j\omega t}. \tag{1.19}$$

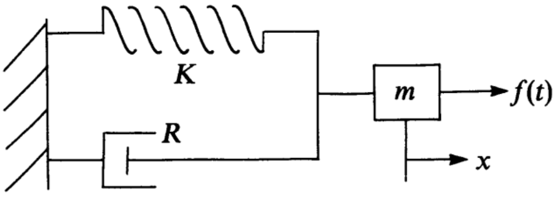


Fig. 1.6. A damped harmonic oscillator with driving force $F(t)$. From reference [2].

The steady-state solution, includes the complex displacement and complex velocity:

$$\tilde{x} = \frac{Fe^{j\omega t}}{K - \omega^2 m + j\omega R} = \frac{\tilde{F}/m}{\omega_0^2 - \omega^2 + j\omega 2\alpha}, \tag{1.20}$$

$$\tilde{v} = \frac{Fe^{j\omega t}}{R + j(\omega m - K/\omega)} = \frac{\tilde{F}\omega/m}{2\omega\alpha + j(\omega^2 - \omega_0^2)}. \tag{1.21}$$

The mechanical impedance which is defined as ratio of force to velocity is another important parameter

$$\tilde{Z} = \tilde{F}/\tilde{v} = R + j(\omega m - K/\omega) = R + jX_m \tag{1.22}$$

In these formulas $\tilde{F} = Fe^{j\omega t}$, $\omega_0^2 = K/m$, and $\alpha = R/2m$.

In many textbooks, instead of presenting the displacement and velocity as complex variables, formulas describing magnitude of displacement (or velocity) and phase difference between displacement (or velocity) and driving force are given for sinusoidally-driven harmonic system These formulas represent magnitude of displacement and phase shown in Fig. 1.7.

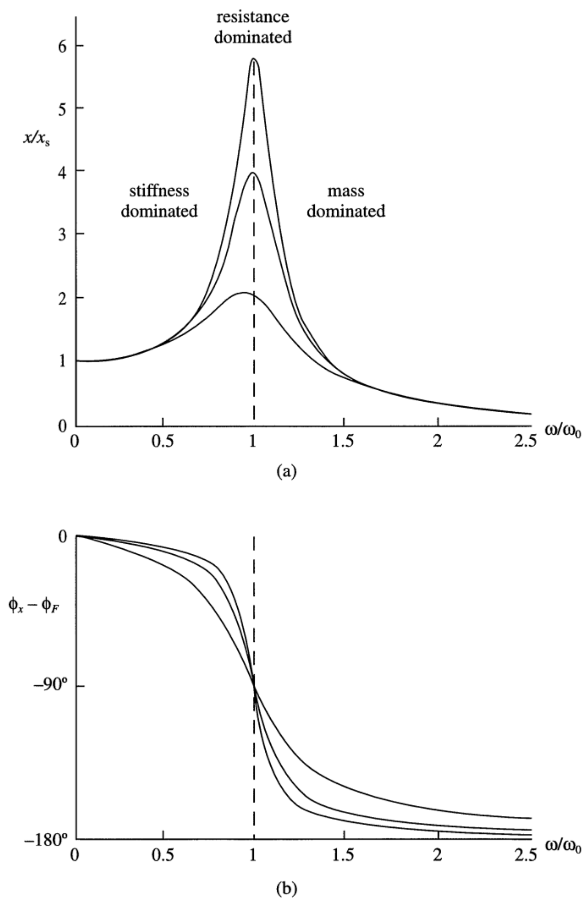


Fig. 1.7. Frequency dependence of the magnitude and phase of the displacement of a linear harmonic oscillator. From reference [2].

For the displacement, a quantity $x_s = F/K = F/m\omega_o^2$ is the static displacement of the oscillator created by a constant force of magnitude F at zero frequency. The displacement amplitude approaches F/K for frequencies decreased down to 0 Hz. In the low frequency range, below the resonance peak, the oscillator is controlled by dominating stiffness. In this case of slow motion, the phase of displacement is nearly equal the phase of the driving force. At the

resonance, when $\omega = \omega_d$ the amplitude becomes

$$x_0 = F/2\alpha m\omega_0 = Qx_s. \quad (1.23)$$

For resonance peak, the Q factor is an amplification factor. The Q is the ratio of the amplitude of displacement at resonance to the static displacement. The system is called resistance dominated since the stiffness of spring is balanced by the mass inertia (reactance equals zero). For high frequencies (above the resonance peak), the displacement falls toward zero and the phase difference between displacement and driving force approaches 180^o. In high frequency range, the system is controlled by the dominating mass. At extremely high frequencies variation of driving force is so fast that mass inertia prevents the system from any displacement.

1.5. Transient response of an oscillator

When a driving force is suddenly applied to an oscillator, its motion is usually complex. The impulsive excitation results in the damped oscillations with frequency $\omega_d < \omega_0$ and exponential decay directly related to the Q factor.

Decrease in amplitude of damped system is shown in Fig. 1.8. According to formulas presented earlier relating Q with angular frequency ω the $Q = \omega_0\tau/2$, where τ is the time required for the amplitude to decrease to $1/e$ (= 0.37) of its initial value.

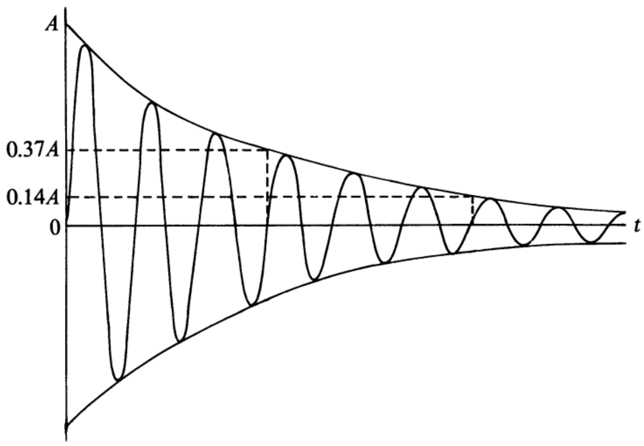


Fig. 1.8. Response of a damped oscillator (Q = 10) to impulsive excitation of a large force The amplitude falls to 37% of its initial value in time τ, which corresponds to Q/n cycles. From reference [2].

When the driving force is periodic initial oscillator response consists of driving force frequency and exponentially decaying oscillations with natural frequency. Transient motion decays rapidly when the oscillator is heavily damped (small Q). For small damping, transient continues during many cycles of oscillation of driving force. For driving frequency close to the natural frequency there are strong phase effects between natural frequency and forced frequency including specific amplitude changes of the envelope and beats. This is shown in Fig. 1.9 for frequency of driving force comparable to natural frequency of the system in comparison with conditions in which driving force frequency is either larger or smaller than the natural frequency.

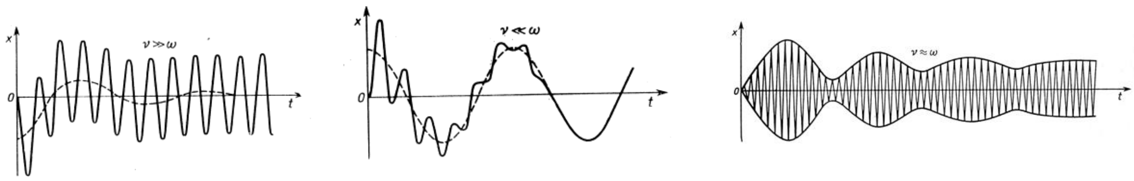


Fig. 1.9. Response of a damped oscillator for natural frequency larger, smaller, and comparable to driving force frequency. From reference [3].

In Fig. 1.10 time waveforms are shown for various ratios of driving force frequency and natural frequency of the system. These waveforms indicate that even for a simple resonance system, the transient excitations can be quite complex.

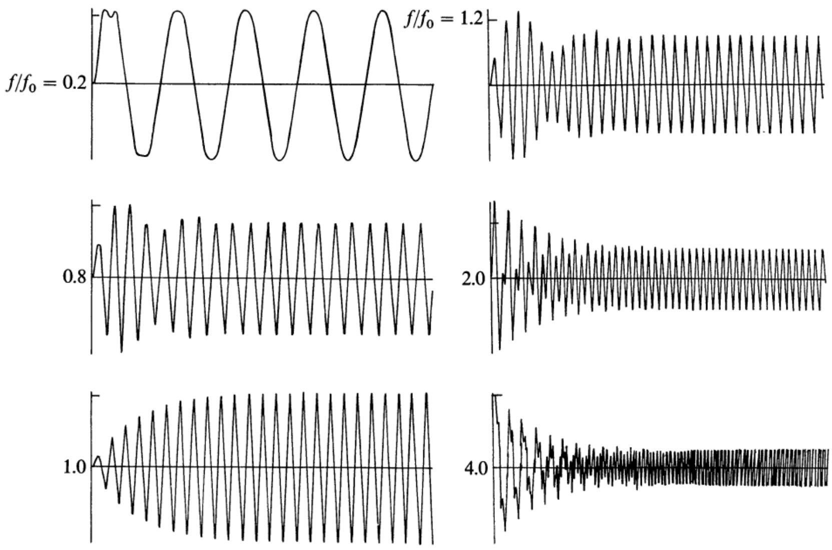


Fig. 1.10. Response of a simple oscillator to a sinusoidal force applied suddenly. The ratio f/f_o of driving force and natural frequencies varies from 0.2 to 4.0, Q = 10. From reference [2].

Time waveforms seen in Fig. 1.9 result from the formula (1.24) below in which the first term represents exponentially decaying transient part of oscillation with frequency ω_d and second term the steady state part of the oscillator response with frequency ω of driving force :

$$x = Ae^{-\alpha t}\cos(\omega_d t + \phi) + \frac{F}{\omega Z}\sin(\omega t + \theta). \quad (1.24)$$

1.6. Acoustic vibrating systems

Some acoustic systems such as a piston of mass m moving with no friction in a cylinder and a Helmholtz resonator are shown in Fig. 1.11.

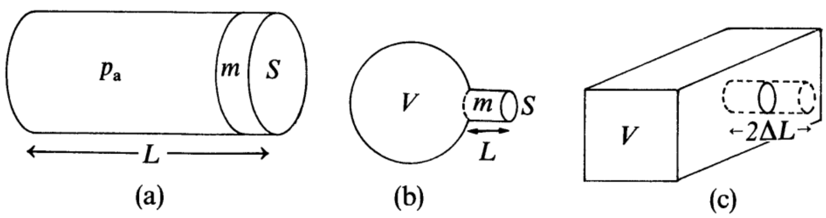


Fig. 1.11.Simple vibrating systems:(a) piston in a cylinder; (b) Helmholtz resonator with neck of length L; (c)

Helmholtz resonator without a neck. From reference [2].

The piston of mass *m* (Fig. 1.11a) which moves in a cylinder of area *S* and length *L* vibrates like a mass attached to a spring. The spring constant is $K = \gamma P_a S/L$ ($\gamma = 1.4$ for air). The natural frequency of the motion is

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{\gamma p_a S}{mL}} \tag{1.25}$$

The Helmholtz resonator shown in Fig. 1.11b is purely acoustical system. It differs from the cylinder shown in Fig. 1.11a in abrupt geometrical change between the neck and the bottle *V*. This causes that the mass of air in the neck constitutes moving mass (there is no changes in pressure in the neck). The large volume of air *V* acts as the spring. This is important property of the Helmholtz resonator that large volume *V* and the neck are filled with the same medium. Unlike that, for cylinder if there is no specific piston constituting the mass other than medium it is hard to define the boundary for the mass. In other words, an empty cylinder constitutes rather a quarter wavelength resonator.

For the Helmholtz resonator, the mass of air in the neck and the spring constant are given by the expressions

$$m = \rho SL, \tag{1.26}$$

and

$$K = \rho S^2 c^2/V \tag{1.27}$$

where ρ is the air density and *c* is the speed of sound.

The natural frequency of vibration is given by

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{K}{m}} = \frac{c}{2\pi}\sqrt{\frac{S}{VL}}. \tag{1.28}$$

In Fig. 1.11c, there is the Helmholtz resonator with no geometrical neck. The effective length comes from the "end correction" taken twice, which is $8a/3\pi = 0.85\,a$ (a is the neck radius). The natural frequency of a Helmholtz resonator with no neck is as

$$f_0 = \frac{c}{2\pi}\sqrt{\frac{1.85a}{V}}. \tag{1.29}$$

1.7. Driving point impedance

A mechanical impedance of simple harmonic system was defined as the ratio of external force to the velocity of oscillation. In complex system, such as vibrating plate or other structure the impedance depends on the place at which the force is imposed and velocity measured. Therefore when the force *F* is applied at a single point and the velocity *v* is measured at this same point, the ratio of *F*/*v* is called the driving-point impedance.

The measurement is often made by means of an impedance head, which consists both a force transducer and accelerometer (Fig. 1.12). In this measurement setup the inertial mass of the accelerometer adds to the structure introducing an error to the measurement; the mass should be mass as small as possible.

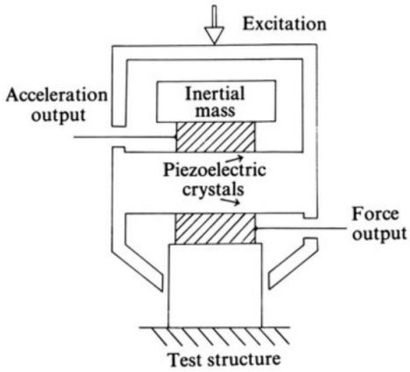


Fig. 1.12. Impedance head consisting of an accelerometer and force transducer. From reference [2].

Traditional measurement setup includes independent electromagnetic shaker, which provides constant force of excitation. The acceleration measured by the accelerometer has to be integrated to obtain a velocity signal of the movement. If the force is constant along the swept frequencies the measured velocity signal is directly proportional to impedance of interest.

2. Sound waves in air

The important point for considering sound wave is that the ear in our hearing process predominantly reacts to pressure variations in air resulting from a soundwave. This places sound pressure in first place among the various acoustical parameters characterizing the acoustic wave propagation. Luckily sound pressure is the easiest parameter to measure.

2.1. Plane wave

Sound waves are the mechanical waves that propagate in any medium that has mass and elasticity. Those include gases, fluids and solids. Gases (e.g. air) and fluids (e.g. water) do not have elastic response to shear, only the response to compression and thus waves that can propagate in gases and fluids are longitudinal, with the motion of molecules being in the same direction as the direction of the wave itself (Fig. 2.13). Solids have both shear and compressive elasticity. Therefore, in solids both transverse (shear) and longitudinal (compressive) waves can propagate each with different speed and directivity. Compressive and shear waves are often explained by the chain of previously discussed spring-mass harmonic motion systems (Fig. 2.14). In general, a sound wave can be treated as the propagation of the local compressions and rarefactions of oscillating molecules. In acoustics, the thermodynamics of molecule motion is usually disregarded. With this view, with no sound wave propagated there is no particle motion in the medium. This is justified by the fact, that sound pressure level of noise associated with Brownian motion is in the range of −30 dB.

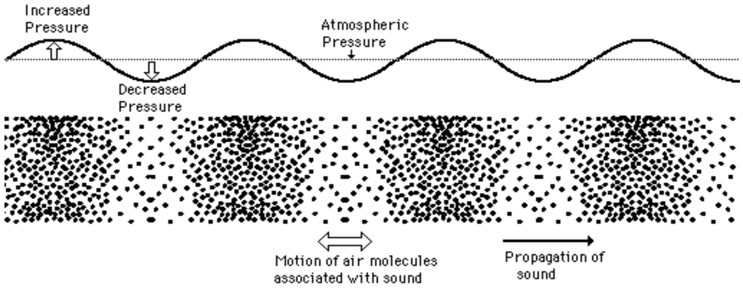


Fig. 2.13. Idealized explanation of longitudinal wave in gas.

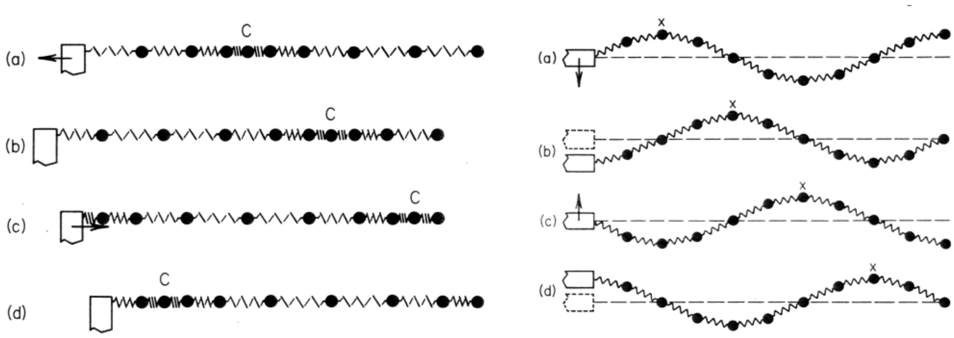


Fig. 2.14. Model of compressive and shear waves by the chain of spring-mass harmonic motion systems.

There are three major types of sound waves considered to represent most of real sound waves with some simplification to have convenient mathematical description. These include plane wave propagating along one direction as it would be generated by an infinite plane surface moving perpendicularly to it, the spherical wave generated by a small pulsating source and spreading out omni-directionally in three dimensions, and a cylindrical wave generated by line of pulsating sources and propagating omni-directionally in two dimensions. It is the simplest to consider plane waves as only one space coordinate *x* has to be taken into account.

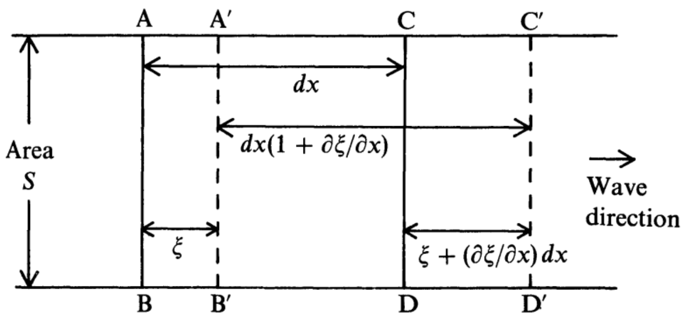


Fig. 2.15. A plane wave: infinitely small volume of medium moving along wave direction (*x*). Element ABCD moves forward by displacement ξ changing its volume to A'B'C'D'. From reference [2].

In Fig. 2.15, the displacement and expansion of the air due to sound wave is represented by element *ABCD* having thickness *dx* and volume *Sdx* which moves to the position of *A'B'C'D'* also increasing its volume by *dV*

$$V + dV = S\,dx\left(1 + \frac{\partial \xi}{\partial x}\right), \qquad (2.30)$$

The volume change by *dV* creates change in pressure p_a by dp_a, which is

$$dp_a = -K\frac{dV}{V} \tag{2.31}$$

In equation (2.31) the proportionality constant K is called bulk modulus and describes to physical compressibility of a medium (either gas or fluid). The small variation dp_a of atmospheric pressure p_a is called sound pressure and denoted by p. Equation (2.30) allows to rewrite Eq. (2.31) as

$$p = -K\frac{\partial \xi}{\partial x} \tag{2.32}$$

Motion of the element *ABCD* must follow the Newton's second law $F = ma$ what incudes the difference in force F along displacement ξ, and acceleration a of mass $VS\rho$ of the medium enclosed by volume V:

$$-S\left(\frac{\partial p}{\partial x}dx\right) = \rho S\,dx\frac{\partial^2 \xi}{\partial t^2}\text{, or}\quad -\frac{\partial p}{\partial x} = \rho\frac{\partial^2 \xi}{\partial t^2}. \tag{2.33}$$

Applying Eq. (2.32) to Eq.(2.33) leads to the one-dimensional wave equation for displacement:

$$\frac{\partial^2 \xi}{\partial t^2} = \frac{K}{\rho}\frac{\partial^2 \xi}{\partial x^2} \tag{2.34}$$

and a similar equation for sound pressure:

$$\frac{\partial^2 p}{\partial t^2} = \frac{K}{\rho}\frac{\partial^2 p}{\partial x^2} \tag{2.35}$$

In general form of wave equation the ratio $K/\rho = c^2$, where c is the propagation speed of sound. Therefore the equation (2.35) can be written in the form

$$\frac{\partial^2 p}{\partial t^2} = c^2\frac{\partial^2 p}{\partial x^2} \tag{2.36}$$

In all these equations, the physical properties of medium are represented by bulk modulus K and density ρ.

In normal conditions, that is not for a very high frequencies, or low frequencies and not close to thermo-conductive materials, the sound wavelengths are long enough to prevent from thermal conduction between points of high and low pressure. This is valid for most common conditions of sound propagation in air. Therefore, the wave behavior is considered adiabatic, and described by

$$p_a V^{\gamma} = \text{constant} \tag{2.37}$$

where p_a is atmospheric pressure and $\gamma = C_p/C_v = 1.4$ is the ratio of the specific heats of air at constant pressure and at constant volume. The sound speed c can be thus expressed as:

$$c^2 = \frac{K}{\rho} = \frac{\gamma p_a}{\rho} \tag{2.38}$$

Sound speed in air noticeably changes with temperature, increasing with temperature. The change of sound speed with temperature follows the equation

$$c(T) = \left(\frac{T}{T_0}\right)^{1/2} c(T_0) \quad (2.39)$$

where T is the absolute temperature. Another relation of sound speed to temperature valid for 50% relative humidity is

$$c \approx 332(1 + 0.00166\Delta T)\ [\mathrm{m/s}] \quad (2.40)$$

where ΔT is in ^{o}C.

Sound speed is about 343 m/s in room temperature and 331 m/s in temperature of 0^{o} Celsius.

Table 2.1 Sound speed in various materials. From reference [1].

Substance	Temperature (°C)	Speed (m/s)	Speed (ft/s)
Air	0	331.3	1,087
Air	20	343	1,127
Helium	0	970	3,180
Carbon dioxide	0	258	846
Water	0	1,410	4,626
Methyl alcohol	0	1,130	3,710
Aluminum	—	5,150	16,900
Steel	—	5,100	16,700
Brass	—	3,480	11,420
Lead	—	1,210	3,970
Glass	—	3,700–5,000	12–16,000

2.1.1. Solutions to plane wave equation

There are two solutions to wave equation:

$$\boldsymbol{p(x,t) = f_1(x - ct) + f_2(x + ct)} \quad (2.41)$$

where $f_1(x - ct)$ represents a wave of an arbitrary spatial shape f propagating in the $+x$ direction at speed c, and $f_2(x + ct)$ a wave of an arbitrary spatial shape f propagating in the $-x$ direction at speed c.

In frequency domain (for sinusoidal excitation), solutions to wave equation have a form that uses complex notation

$$p = Ae^{-jkx}e^{j\omega t} + Be^{jkx}e^{j\omega t}, \quad (2.42)$$

where $k = \omega/c = 2\pi/\lambda$ is a wave number equivalent in its role along distance x to angular frequency ω in time. The $\lambda = cT = c/f$ is the wavelength corresponding to the period $T = 1/f$.

For wave of angular frequency ω traveling in the $+x$ direction the equation has the form:

$$\boldsymbol{p = e^{-jkx}e^{j\omega t} \rightarrow \cos(\omega t - kx).} \quad (2.43)$$

Complex notation on the left represents its real part shown on the right.

2.1.2. Wave impedance

Assuming that $p = e^{-jkx} e^{j\omega t}$ and also that ξ must have similar form but different amplitude we can calculate by differentiating p in x dimension, and ξ in time t (the equation (2.33)). Then, the relation between sound pressure p and particle velocity u is

$$jkp = j\rho\omega\frac{\partial\xi}{\partial t} \tag{2.44}$$

and finally

$$p = \rho c u \tag{2.45}$$

where u is the particle velocity.

The wave impedance (or specific acoustic impedance) is for plane wave a real quantity:

$$z = \frac{p}{u} = \rho c \quad , \tag{2.46}$$

Wave impedance is only related to the physical properties of medium: density ρ and sound speed c. Thus for plane wave, the acoustic pressure and particle velocity in the propagation direction are in phase: plane wave creates best conditions for transport of energy with the direction of wave propagation. This relation is clearly seen in Fig. 2.16. Particle displacement ξ, in Fig. 2.16 part (a), creates changes in density of medium molecules (b) and thus an increase or a decrease in pressure. Differentiating particle displacement to obtain the particle velocity u in (c) creates phase shift in 90^o (particle at maximum displacement is at zero velocity). Therefore pressure shown in panel (d) is in phase with velocity u in panel (c).

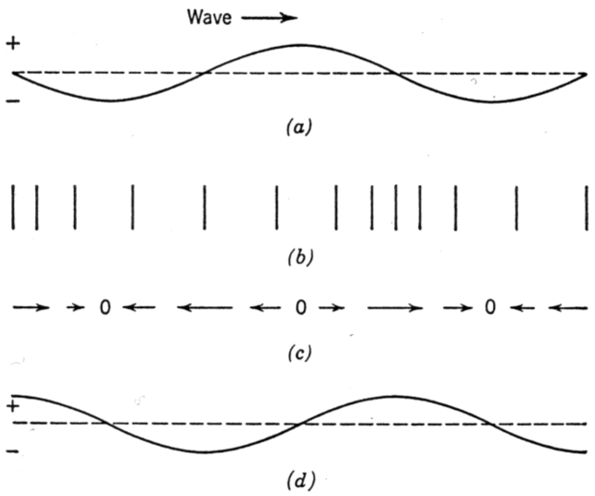


Fig. 2.16. A plane wave traveling in the positive x dimension. (*a*) Displacement ξ as a function of position. (*b*) Spacing of particles displaced accordingly to (*a*). (*c*) Particle velocity. (*d*) Pressure and condensation. From reference [4].

The units of wave impedance are Pa m^{-1} s (kg m^{-2} s^{-1}). This unit is called *rayls* (after Lord Rayleigh - John Strutt (1842−1919)). Wave impedance changes with temperature:

$$z = \rho c \approx 428(1 - 0.0017\ \Delta T)\ \text{kg m}^{-2}\ \text{s}^{-1} \tag{2.47}$$

where ΔT is in degrees Celsius. For air at temperature 0°C and standard pressure the wave impedance of air equals 428 *rayls*. For a comparison, wave impedance of fresh water is 3560 times greater than for air.

2.2 Spherical wave

Wave equation for the spherical wave is typically treated in polar coordinates. This equation can be found in suggested readings. Here, only the solutions to it will be discussed. General solution for pressure *p* in spherical wave is a superposition of two waves: wave outgoing (-*jkr*) and wave incoming (+*jkr*) to the origin of the disturbance

$$p = \left(\frac{A}{r}e^{-jkr} + \frac{B}{r}e^{jkr}\right)e^{j\omega t}. \tag{2.48}$$

The acoustic particle velocity *u* for the outgoing wave (only for *A*, *B* = 0) is

$$u = \frac{A}{r\rho c}\left(1 + \frac{1}{jkr}\right)e^{-jkr}e^{j\omega t} \tag{2.49}$$

There are two components of the particle velocity *u* in spherical wave. The far field component is in phase with pressure change. It falls off over distance with $1/r$. The near field component is 90° phase shifted and decreases with $1/r^2$ over distance. These components are called far and near field, respectively, as the $1/r^2$ component is particularly large close to the origin but quickly diminishes with distance. The $1/r$ far field component is dominating at large distances from the wave origin. The far field *u* component is related to pressure *p* through ρc wave impedance exactly as it was for the plane wave (but both pressure and particle velocity decrease with distance with $1/r$). Near and far field components are equal to one another at a distance of about 0.16λ (Fig. 2.17).

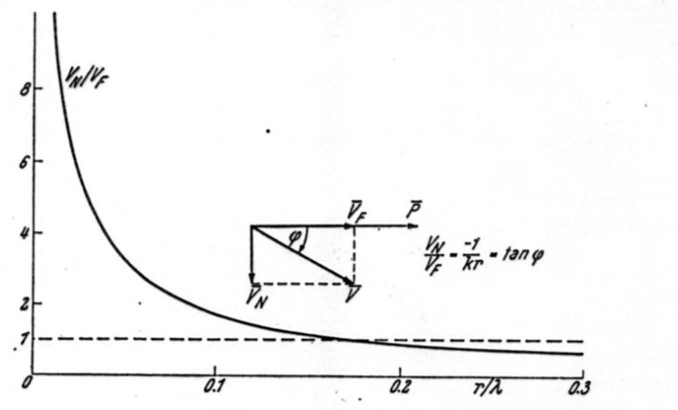


Fig. 2.17. Relation between nearfield and far field particle velocity components with distance for spherical wave. From reference [4].

Ratio of pressure and particle velocity gives the wave impedance for spherical wave which depends on distance from the origin of the wave:

$$z = \frac{p}{u} = \rho c\left(\frac{jkr}{1 + jkr}\right). \tag{2.50}$$

Near the origin, when $kr \rightarrow 0$ wave impedance is much smaller than ρc. It approaches ρc at large distances.

2.3 Sound intensity

Sound intensity is the vector quantity to describe the flow of the acoustic energy carried forward with the wave. Its value is defined as a sound power per unit area of surface perpendicular to the direction of sound propagation. As a vector sound intensity points at the direction of propagation. The acoustic intensity ***I*** is measured in watts per square meter. For progressive wave, sound intensity increases with sound pressure. For standing wave, the sound pressure may be large, but the intensity remains small since the energy is not transported with the wave. Energy that is temporarily stored in the medium, either moving back and forth or stored in mass reactance close to the surface of a sound source (discussed later) does not contribute to the sound intensity.

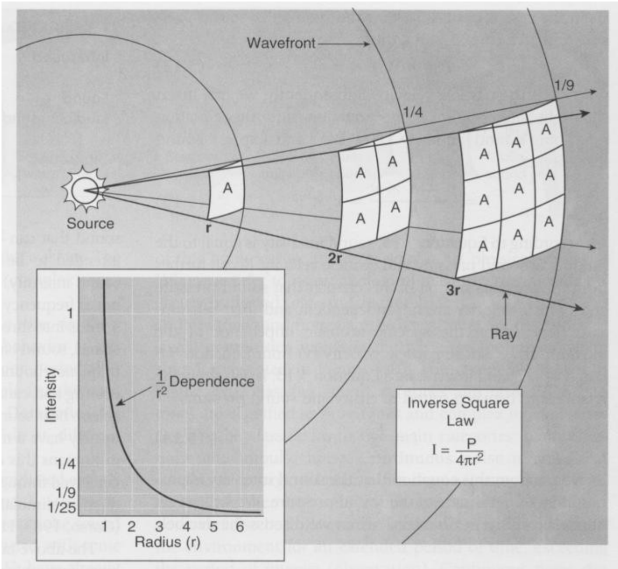


Fig. 2.18. Graph explaining change in sound intensity with distance from the origin for spherical wave. From reference [1].

For a plane wave, the sound intensity is calculated just like for power from voltage and current being in phase in electric circuits

$$I = \rho c u^2 = \frac{p^2}{\rho c} = pu, \tag{2.51}$$

where p and u are effective **rms** quantities.

For spherical wave, large part of the energy in the nearfield (close to the origin) is not radiated because $1/jkr$ factor creates 90° phase shift of particle velocity relative to the sound pressure. The radiated intensity is related to the far field component of the velocity in Eq. (2.49), and sound intensity decreases with $1/r^2$ (Fig. 2.18).

The total power P radiated with a spherical wave can be calculated by integrating $I(r)$ over a spherical surface of radius r, giving

$$P = \frac{4\pi r^2 p(r)^2}{\rho c} \tag{2.52}$$

A source radiating a power of 1 mW as a spherical wave produces an intensity level, or equivalently a sound pressure level (SPL), of approximately 79 dB at a distance of 1 m. At a distance of 10 m, assuming no reflections from surrounding walls or other objects, the SPL is 59 dB. Decibel scale for sound power is constructed in similar way as for sound intensity with reference power of 10^{-12} Watts.

For an ideal plane wave the surface perpendicular to the direction of wave propagation is assumed infinite. Thus, calculating total sound power level for plane wave is not possible (sound power in ideal case would be infinite).

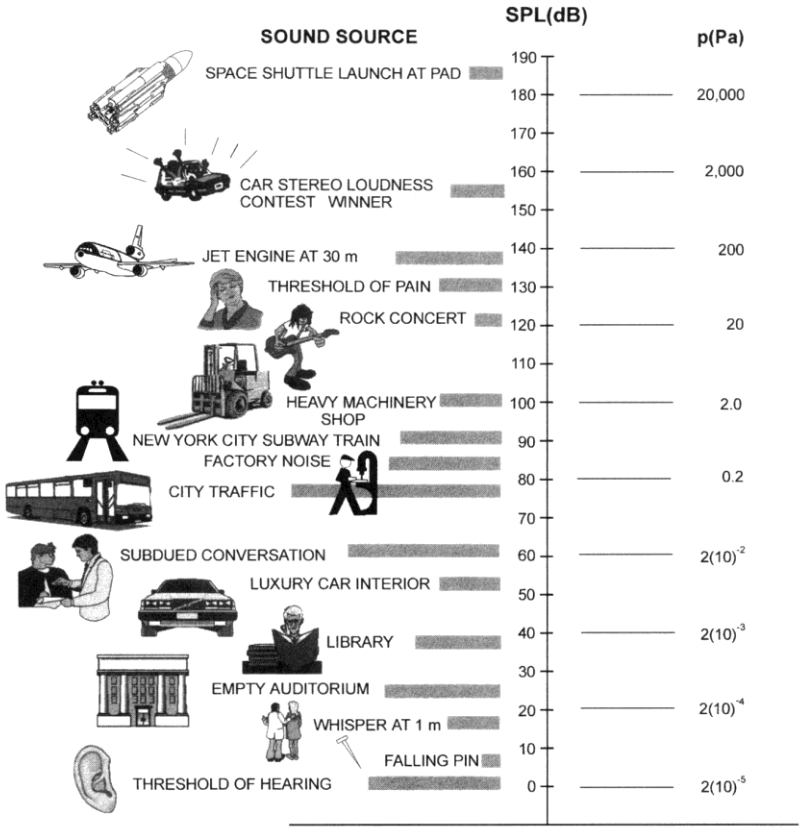


Fig. 2.19. Sound pressure levels and examples of corresponding everyday sounds. From reference [5].

2.4 Decibels: Sound intensity level and sound pressure level

Decibel scale in acoustics is very common as human ear takes logarithm of sound pressure to create sensation of loudness which was first expressed by the Weber–Fechner's law formulated by Fechner in *Elemente der Psychophysik* in 1860. The intensity level is defined as

$$L_I = 10\log_{10}\left(\frac{I}{I_0}\right) \tag{2.53}$$

in decibels. The factor 10 creates conversion from basic unit of bell (after Graham Bell) to decibel. The reference intensity I_0 is selected as 10^{-12} W/m^{-2}, which is selected as a simple number (meaning that only integer exponent was selected), providing sound intensity close to the threshold of human hearing in the frequency range from 1 to 3 kHz.

The sound pressure level (SPL) is defined as

$$L_p = 20\log_{10}\left(\frac{p}{p_0}\right) \tag{2.54}$$

with reference sound pressure of 20 μPa selected to correspond to I_0 (p_0 is the square root of I_0 multiplied by ρc; in numbers p_0 equals square root of $412{\cdot}10^{-12}$). For plane wave in air the dB scales for sound intensity and sound pressure are identical. For p and p_0 rms values are used because of relation to sound intensity $I \approx p^2$. On the SPL scale, 1 Pa corresponds to about 94 dB. Examples of sound pressure levels for everyday sounds are given in Fig. 2.19.

2.5. Sound reflection and transmission

There are several phenomena which occur in media whose dimensions are limited, namely reflection, transmission, refraction, absorption, and diffraction (Fig.2.20).

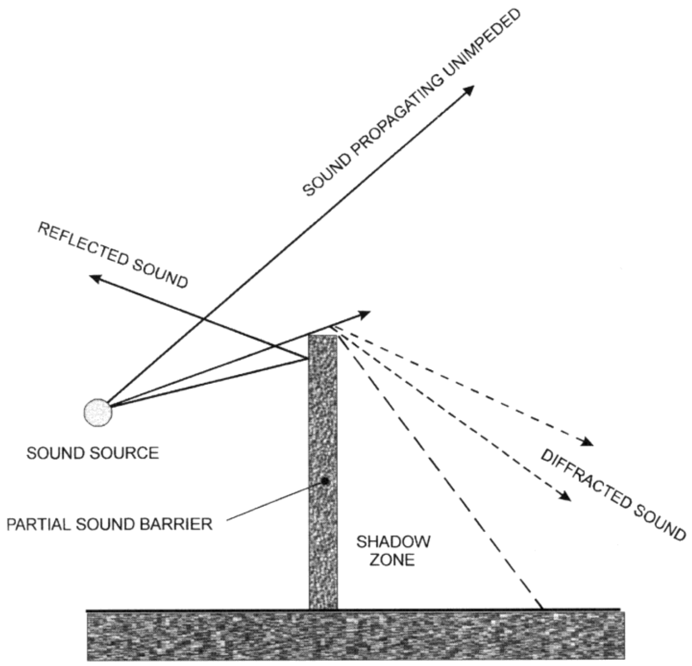


Fig. 2.20. Basic changes in sound propagation for incident wave hitting an object: reflection, diffraction and acoustic shadow. From reference [5].

When a wave passes variations in the properties of the medium its propagation is disturbed. Gradual changes in the medium lead usually to a change in the speed and direction of propagation, which is the phenomenon of refraction. When there is an abrupt but smooth boundary between the two media (Fig. 2.21), an example of which is the surface of a see or a lake, the acoustic wave partly undergoes specular reflection and partly refracts according to the Snell's Law of geometrical optics

$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{k_1}{k_2} = \frac{c_2}{c_1} \tag{2.55}$$

where indexes 1 and 2 refer to the first and second medium (incident and refracted waves), respectively, k_1 and k_2 are the wave numbers and c_1 and c_2 the sound speeds. This relation imposes obvious property that the frequency of excitation remains the same in both media. As for optics Θ_1 and Θ_2 are the angles with respect to the normal to the boundary. In addition, in such a case angle Θ_3 of a reflected sound equals incident angle Θ_1 because of specular reflection.

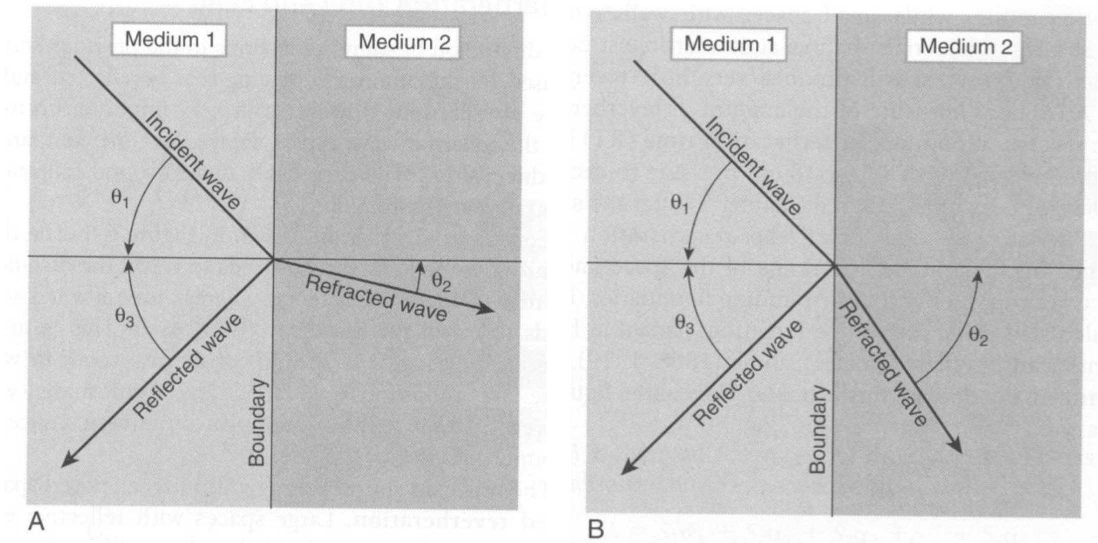


Fig. 2.21. Sound refraction at the boundary between two media: A. $c_2 < c_1$. B. $c_2 > c_1$. From reference [1].

It is to note that except for purely technical applications that usually only occur for ultrasound, the sound speed in most media surrounding us in real life is greater than that in the air. The first at hand evident example is see water or fresh water. The sound speed in the water is about four to five times greater than that in the air (343 m/s in air versus 1484 or 1500 m/s – fresh or see water). Therefore for any angle of incidence Θ_1 different from 0^o, the Θ_2 in the water is larger. There is a critical Θ_1 value for which the $\Theta_2 = 90^o$ meaning the so called total internal reflection. A total internal reflection, which is very common in optics, results in inability for a soundwave to enter the new propagation medium. For sound entering the water from the air, critical Θ_1 can be calculated as follows:

$\sin\Theta_2 / \sin\Theta_{1crit} = 1484/343 = 4.32 \rightarrow \sin\Theta_{1crit} = 0.23 \rightarrow \Theta_{1crit} = 13.36°$.

The value tells us that when the angle of incidence of sound wave approaching water is further away from normal to the surface than about 13° the soundwave is reflected in full from the water surface. This kind of phenomenon can be generally observed for sound in air confronted with boundaries of other media as for most substances the sound speed is significantly larger than that in the air (see Table 2.1). Thus, an important condition to consider is the reflection of sound approaching media boundary at normal angle. The purpose is to estimate the amplitude of sound pressure and particle velocity of reflected and transmitted sounds with respect to the incident sound.

For acoustic wave, the two media are different when their wave impedances z_1 and z_2 differ from each another. Analysis of reflected and transmitted waves is commonly presented in a simple way in textbooks. Assume that incident pressure plane wave is Ae^{-jkx} moving in plus x direction with amplitude A. Similarly the reflected wave of amplitude B can be written as Be^{jkx} as it travels in minus x direction. The transmitted wave is represented by Ce^{-jkx}. The problem to solve is to find ratios B/A and C/A of amplitudes thus to refer transmitted and reflected waves to the incident wave. The boundary conditions require that the pressure on the one side (for incident combined with reflected wave) is equal to the pressure at the other side (for transmitted wave)

$$A + B = C. \qquad (2.56)$$

Similarly sum of particle velocities at the incident side has to be equal particle velocity at the transmitted side. According to the definition of wave impedance, for pressure amplitudes A and B, the plane wave particle velocities are A/z_1 and B/z_1, and at transmitted side C/z_2 thus

$$\frac{A - B}{z_1} = \frac{C}{z_2}, \qquad (2.57)$$

where minus sign comes from the change of the travel direction of reflected wave.

A combination of the two equations leads to the ratios *B*/*A* and *C*/*A* of amplitudes

$$\frac{B}{A} = \frac{z_2 - z_1}{z_2 + z_1}\,, \tag{2.58}$$

and

$$\frac{C}{A} = \frac{2z_2}{z_2 + z_1}\,. \tag{2.59}$$

Similar equations developed for the sound intensities are for reflected sound:

$$\frac{I_r}{I_0} = \left(\frac{z_2 - z_1}{z_2 + z_1}\right)^2\,, \tag{2.60}$$

and transmitted sound:

$$\frac{I_t}{I_0} = \frac{4z_2 z_1}{(z_2 + z_1)^2}\,, \tag{2.61}$$

where I_0, I_r, and I_t are respectively sound intensities of the incident, reflected, and transmitted waves.

It is possible to derive some important properties of reflection and transmission from equations (2.58) - (2.61). If $z_2 = z_1$, then B = 0 and C = A. Therefore, there is no change in transmission to a new medium, there is no new medium in terms of the sound wave propagation. In practice, this is also a case when there is a small difference in wave impedances, for $z_2 \approx z_1$. For $z_2 > z_1$, the reflected wave is in phase with the incident wave (a pressure maximum reflects as a pressure maximum). For $z_2 < z_1$, there is a phase inversion of the reflected wave and a pressure maximum is reflected as a minimum. For $z_2 \gg z_1$ or $z_1 \gg z_2$, the reflection is almost total. In practice, even a tenfold difference in wave impedances produces strong reflection, as B/A = 0.9/1.1 = 0.81 (in the example arbitrary $z_2 = 1$, $z_1 = 0.1$). Equation (2.58) shows that for $z_1 \gg z_2$ the transmitted wave is of doubled pressure amplitude. This is not related to any transmitted wave, however, since the particle velocity in such case in medium z_2 equals zero. This is clearly seen in equations (2.60) and (2.61) for sound intensities.

It should be noted that the conditions for a boundary with solids are different. For most angles of incidence, energy of the incoming longitudinal sound waves in gas or fluid is in part converted to longitudinal and in part to transverse waves at the boundary. Only in case of isotropic solid medium and normal incidence propagation remains longitudinal in the solid, and equations (2.58) to (2.61) can be applied. For other angles of incidence particle velocity vector component parallel to the boundary surface ‘slides’ along it and produces transverse wave in solid medium. Analysis of this phenomenon is very complex.

As it was mentioned, the presented reflection/transmission rules are valid for large and flat boundary surfaces. For small objects that are smaller than the sound wavelength (size up to 10 times the wavelength) the wave approaching scatters in all directions. So called acoustic shadows behind obstacles are complementary to reflection and scattering of incoming wave. Very large objects compared with the wavelength produce observable shadows.

Objects of size comparable to the wavelength due to diffraction around the edges diminish or entirely eliminate acoustic shadow at distances comparable to objects diameter. It is important for sound waves propagation that for a given object, the shadow can be noticeable for high audible frequencies but not existing in low frequency range. This is because the wavelengths for the sound propagating in the air changes from about 3.43 meters down to 3.43 centimeters for increase in frequency from 100 Hz to 10000 Hz. Sizes of most things surrounding us fits these dimensions. Set of chairs may be scattering for waves in lower frequency range but their flat seats and back plates may provide specular reflection for frequencies above 5 kHz.

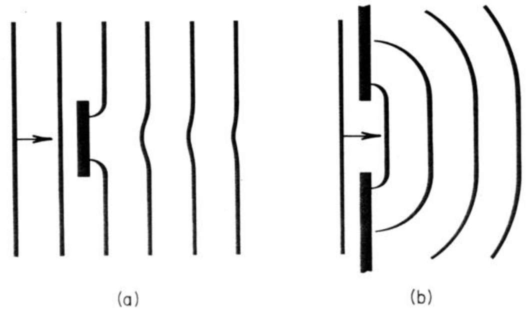


Fig. 2.22. Diffraction and acoustic shadow: a) around the obstacle b) through the aperture From reference [6].

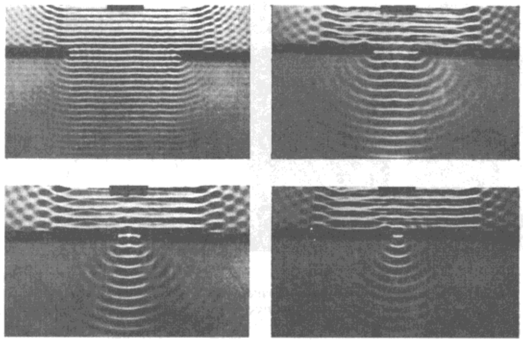


Fig. 2.23. Diffraction of plane wave on water behind the apertures of different size. Exemplification of the Huygens principle. From reference [6].

The diffraction of soundwaves follows the well-known Huygens (1629-1695) principle which later found mathematical formalization in diffraction equations by Fresnel (1788-1827) and Kirchoff (1824-1887). The principle states that any point at which a wave arrives becomes a source of the spherical wave. In non-disturbed conditions such as for the middle part of largest apertures in Fig. 2.22b and in photographs shown in the upper left panel of Fig. 2.23 the sum of spherical waves simply restores the plane wave in the next position of propagation. In contract, at any edges at the obstacles or apertures the diffraction in a form of partial spherical wave can be observed. For small apertures in Fig. 2.23 the spherical wave front at the output of the apertures exemplifying the Huygens principle is clearly seen.

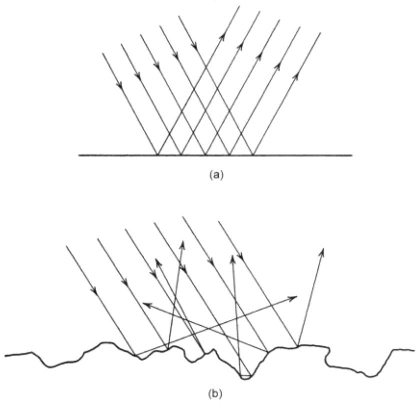


Fig. 2.24. Exemplification of (a) specular, mirror-like reflection, and (b) scattered reflection. From reference [12].

In contrast to specular reflection, scattered reflection regardless of the angle of incoming sound produces waves reflected at random angles (Fig. 2.24). Scattering results from reflecting from porous surfaces. For light, essentially all surfaces, except those especially polished for a mirror-like effect are a source of scattered reflection. This makes it possible for us to normally see almost all objects in the environment. Diffraction and scattering of sound is complex due to the range of wavelengths that correspond to audible range of frequencies. A surface of *x* by *y* dimensions 1 by 1 meter can be perfectly reflecting in high frequency range in which the wavelength is of few centimeters or less, or diffraction will dominate for sound wave of 1 kHz or less for which the wavelength is 0.3 to more than 3 meters long. Common objects or structures like, for instance, a staircase can provide specular reflection as a large object (no steps visible by sound) in low frequency range (less than 100 Hz), specular reflection from a surface of each step in high frequency range (3 kHz and above), or cause a scattered reflection for wavelengths corresponding to mid frequency range. Scattered reflection is an important phenomenon for concert halls and will be treated in a greater detail in room acoustics section.

2.6. Sound absorption

Absorption is related to attenuation of sound in a medium due to dissipation of acoustic energy by of various kinds of processes. Second usage of this term is for the absorption of energy occurring during reflection of a wave from a surface (e.g. a wall). This is a major issue in room acoustics as this kind of losses is dominating and will be treated in the room acoustics section.
Sound wave is attenuated as it propagates because an element of the air compressed in the wave changes its shape since the compression occurs along one dimension, the direction of propagation. The shape change dissipates part of the energy through viscosity among particles which is small but noticeable. There are also thermal effects due to local temperature rise that follows the compression. Heat can be conducted from the warmer compressed parts to the cooler expanded parts of gas. Next reason is that gas such as air is not an ideal gas. Oxygen and nitrogen molecules rotate and vibrate, and sound wave energy transfers to the modes of molecular movement unrelated to the propagation of the sound wave. These are so called molecular losses.

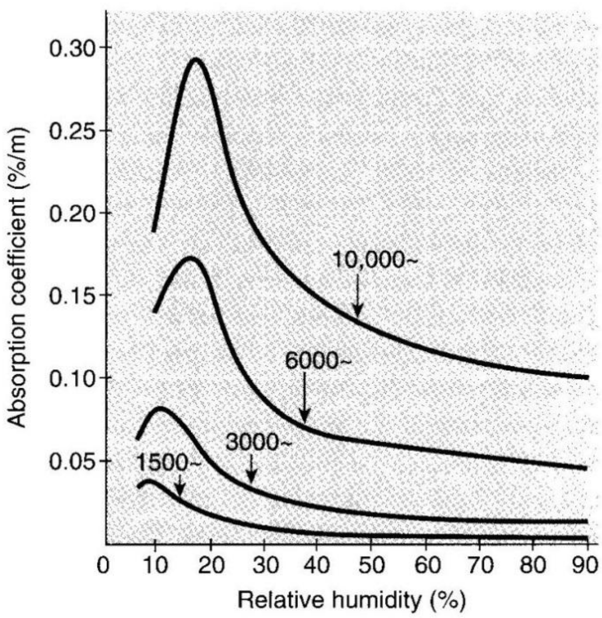


Fig. 2.25. Absorption of sound in air at 20^{o}C. From reference [1].

The amplitude decays during the propagation of a sound wave is represented by term $e^{-\alpha x}$ in propagation equations by introducing the attenuation coefficient α. This is the only factor decreasing the amplitude in a plane wave. In spherical wave, absorption decreases the amplitude as an additional factor to $1/r$ decrease related to distance. As the intensity is proportional to pressure squared the decay of sound intensity due to absorption is $e^{-2\alpha x}$ and for spherical wave it is and additional factor to decrease corresponding to $1/r^2$ due to the geometry of the wave. The attenuation coefficient α is frequency dependent, and $\alpha(f) \approx \omega^2$. This is because the viscous and thermal losses both increase with f^2.

For dry air, the sound absorption is small. With an increase in humidity water vapor molecules through their collisions with air oxygen and nitrogen molecules support transfer of acoustic energy to the vibration modes of molecules causing the attenuation of sound over a wide range of frequencies. For 10 kHz (Fig. 2.25), the attenuation is the highest for about 20% relative humidity (at 20^o C). Lower frequencies are less attenuated but the pattern with peak within 10- 20% of relative humidity remains the same.

Large attenuation of high frequencies by air absorption is significant for propagation of sound over large distances, specifically outdoors. Indoors, the attenuation during propagation can be ignored in small rooms but not completely in very large rooms. Atmospheric absorption amounts to about 0.1 dB/m at 10 kHz (see Fig. 2.26). At a distance of 50 m, it results in a decrease in level by as much as 5 dB.

Figs. 2.26 and 2.27 allow for a comparison of how the sound is absorbed in the air and in the water. The difference is large. For see water, attenuation at 10 kHz is 1 dB per kilometer whereas it is 100 dB in the air. For fresh water the attenuation is nearly two orders of magnitude smaller than that for see water (about 0.003 dB/km). This is because or magnesium salt and boric acid present in see water. They also cause that in see water absorption is not a straight line in log frequency – log absorption coordinates as it is for fresh water.

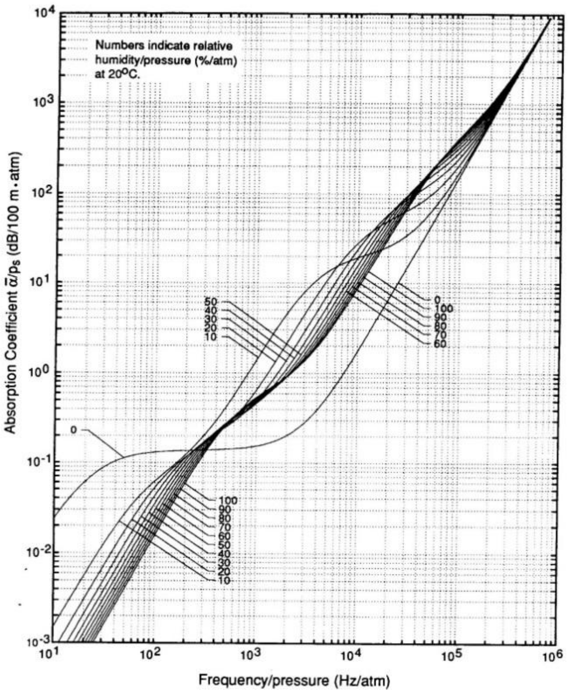


Fig. 2.26 Absorption of sound in the air for various values of relative humidity. From reference [7]

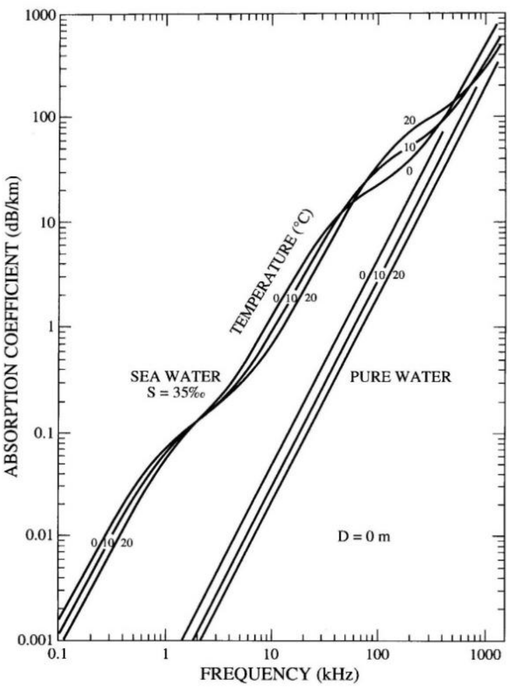


Fig. 2.27. Absorption of sound in fresh and see water at depth 0 m. From reference [7]

2.7. Temperature effects

There are also other factors such as wind, turbulences, and temperature change that strongly affect propagation of sound wave. These effects are most important for outdoor long-distance propagation of sound in the atmosphere.

Normal temperature conditions are associated with the decrease of temperature with height. This decrease depending on the type of weather is about 0.5−1.5 °C per 100 m. Normal temperature gradient causes upward refraction of sound wave. This is because of gradual increase of sound speed with temperature and thus the height. Normal weather conditions are not maintaining long sound propagation at the ground. If only bending of wave rays were considered as it is seen in Fig. 2.28 then sound shadow zones with no sound would occur away from the sound source at the ground. This is not the case due to dispersion on air turbulences which is discussed later.

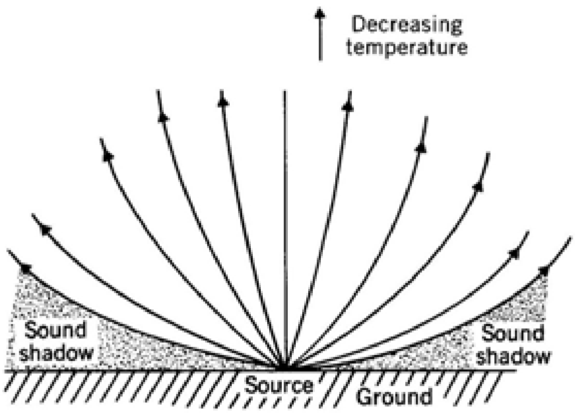


Fig. 2.28. Sound refraction resulting from normal temperature gradient. From reference [7].

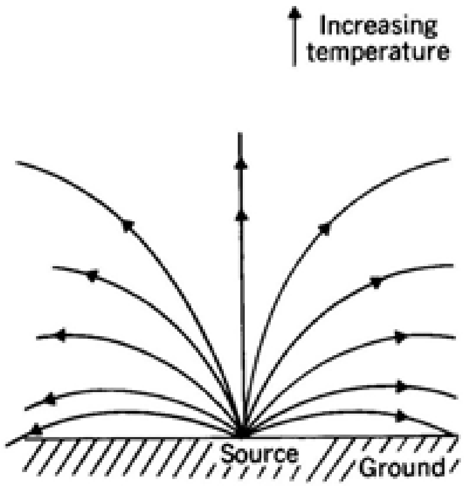


Fig. 2.29. Sound refraction resulting from inverse temperature gradient. From reference [7].

Temperature inversion creates conditions in which there is an increase in temperature with height (Fig. 2.29). Such temperature conditions cause downward refraction of sound wave. Again, this is because of changes in sound speed, in this case gradual increase of sound speed with the increase in temperature and thus the height. Inversion creates conditions for long distant propagation of sound especially when downward bending of sound waves is supported by consecutive reflections from flat surface, such as water. Temperature inversion is most common early at morning and in the evening when sun rise or sun set differently heats various layers of the atmosphere. Other plausible conditions for temperature inversion occur when warm layer of weather front slides up on top of the cold air at the ground. Since such a front is usually associated with rainy weather to come people used to consider good hearing of distant objects as a predictor of rain. It should be noted that temperature inversion only occurs up certain height over ground. Actual sound propagation is as it is shown in Fig. 2.30.

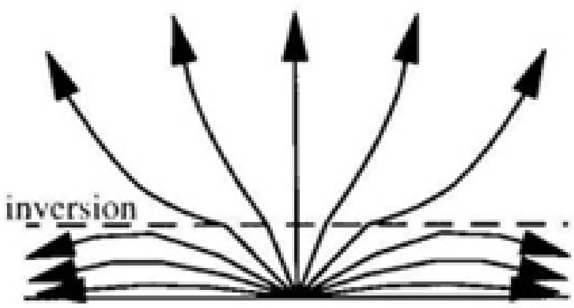


Fig. 2.30. Sound refraction resulting from partial temperature inversion. From reference [7].

2.8. Effects of wind

Effects of wind are similar to temperature effects with respect to bending sound rays upwards or downwards. However, actual mechanism is different. Wind increases its speed with height from few meters per second at the ground to over 10 m/s at 100 m over the ground (in normal weather conditions). Sound propagation is affected as it is shown in Figs. 2.31 and 2.32: propagation is limited in direction against the wind, and strengthened at the surface in the direction with the wind.

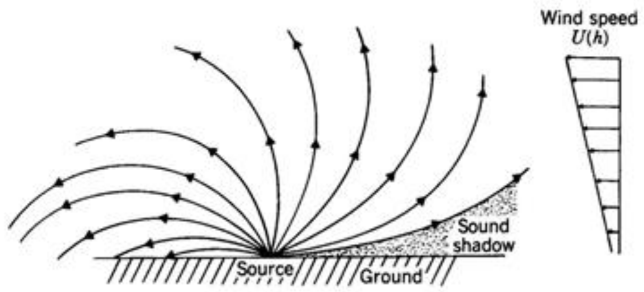


Fig. 2.31. Sound rays resulting from wind blowing with increasing velocity with height. From reference [7].

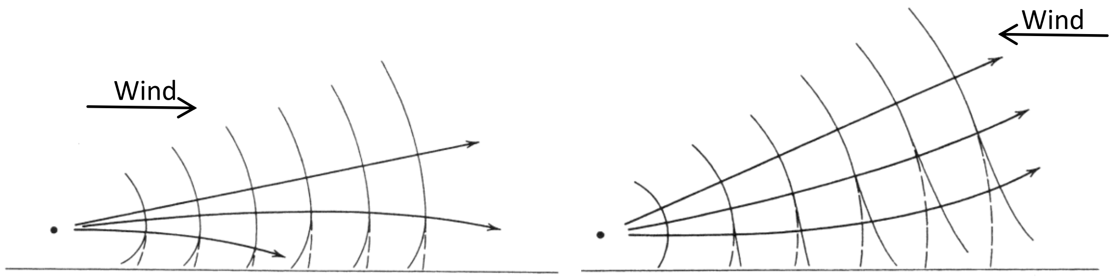


Fig. 2.32. Change in sound wave front shape for propagation with the wind and against the wind. From reference [12].

In the case of propagation with the wind soundwave is tilted toward the ground and form a king of a propagation channel at the surface. Bending of sound during propagation against the wind forms a wave front uniformly tilted upwards.

2.9. Interaction with the ground

Another factor worth mention is the interaction of sound wave along the ground (at normalized height of 4 m). This interaction is significant and is carefully treated in mathematical models of propagation in which flow resistivity of the ground surface is taken into account. For instance, a 7-cm layer of fresh snow will attenuate sound by more than 25 dB in frequency range of about 300 Hz at distance of 15 m from the sound source. Frozen ice surface has no additional attenuation. Grass attenuates sound at frequencies of 500-2000 Hz with noticeable effect at distance of 100 m and more. The influence of surface on wave propagation is carefully treated in acoustics in addition to geometry of the wave and absorption of wave in the medium.

2.10. Diffraction on turbulences

The atmosphere as a medium is characterized by constant irregularities coming from local differences in temperature, pressure, wind conditions, humidity and presence of other substances such as water droplets in foggy conditions. These changes cause sound wave diffraction during propagation which is associated with random changes in propagation speed, and direction.

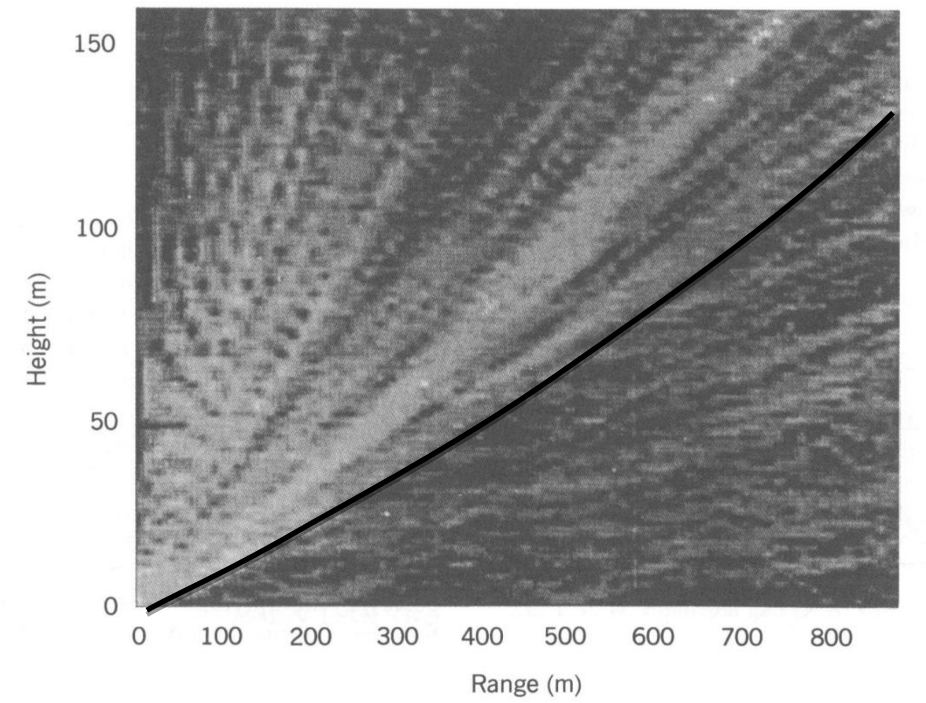


Fig. 2.33. Change in sound wave front shape for propagation with the wind and against the wind . From reference [7].

There has been research mostly based on numerical simulations and modelling showing that diffraction on turbulences is the main cause for lack of acoustic shadow seen in examples of propagation in Figs. 2.28 and 2.31 expected from wave propagation predicted from gradual average changes in temperature or wind speed. Diffraction on turbulences leads to sufficiently significant changes in propagation direction so the acoustic shadow zones are not silent zones but there are merely observed some decrease in level. Fig. 2.33 shows an output from numerical simulations in which lighter points represent sound wave propagation. A line in black delimitates shadow zone at its bottom into which sound rays enter due to diffraction on turbulences. If there were no atmospheric turbulences incorporated into the model but prediction would be based on models related to temperature changes or wind this area would be a silent shadow zone.

2.11. Doppler effect

Doppler effect (Christian Doppler, 1803-1853) was originally introduced to explain change in the emitted light in astronomy (blue shift or red shift) resulting from movement of stars. It is very noticeable effect in sound, especially today when we travel in fast cars and trains as well as we use fast vehicles equipped with sound sources. These vehicles travel is at speeds which are sizable fraction of the sound speed. The effect has found various significant technical applications in detecting motion of objects based on observed frequency shift. One very well-known application is the police radar to control speed of vehicles on the road. In acoustics, the most prominent applications are in ultrasonography for medical purposes, like measurement of blood flow velocity in veins and arteria by Doppler ultrasonography.

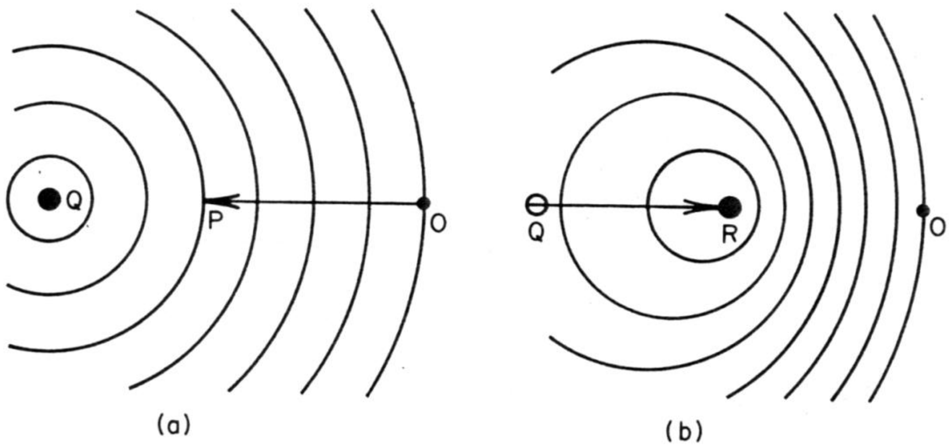


Fig. 2.34. Doppler effect: (a) Observer moves toward the still sound source. (b) Sound source moves at some speed towards still observer.

Doppler effect is the shift in the frequency of observed signal resulting from a movement with velocity *v* of the signal source *S* or the observer *O*. To notice the change in frequency the distance between source and observer must be changing in time otherwise two different Doppler shifts compensate each other. The process is very different for the case when observer moves and for the case in which the source moves although the resulting equations are quite similar.

When the source stays in the same location and the observer moves there is an apparent change in frequency (not related to actual wavelength) seen only by moving observer due to the increase in his relative speed sound. In this case, the apparent increased speed sound is the sum of the speed sound in the medium and the observer's speed of movement. This causes the apparent change in period of sound and received frequency. The acoustic field produced by the source remains unchanged (Fig. 2.34a).

When the source moves entire acoustic field produced by the source is modified with the wavelength in front of the source shortened and enlarged behind the source (Fig. 2.34b). Therefore moving source has different directional frequency characteristics than this source standing still. The major reason for such a picture is that the propagation speed of sound in the medium is independent from any movement of source and depends only on the physical properties of medium. Moving source chases the propagated wave what results in actual shortening the wavelength in front of the source. In a similar way, the sound source moves away from the wave propagated into direction opposite to the direction of movement. This results in actual decrease of the wavelength. In effect, all observers in front of the moving source face the sound propagated with shortened wavelength and experience real increase in frequency. All observers behind the moving source face the sound propagated with enlarged wavelength and experience decrease in frequency. As it was said earlier source movement is the way to change the directional frequency characteristics of the sound source. It is sometimes used by violinists who move the instrument back and forth to produce some impression of vibrato (create frequency modulation).

For the case of observer moving with velocity v_{obs} towards a sound source the apparent sound speed equals sum of sound speed *c* and v_{obs} resulting in higher frequency received by the observer:

$$f_{obs} = (c + v_{obs})/\lambda \qquad (2.45)$$

where $\lambda = c/f_{sc}$ therefore

$$f_{obs} = ((c + v_{obs})/c)\cdot f_{sc} \tag{2.46}$$

Similarly when the observer moves away from the source

$$f_{obs} = ((c - v_{obs})/c)\cdot f_{sc} \tag{2.47}$$

It is worth to note, that with for the same v_{obs} observer moving away from the source observes larger change in frequency and thus larger pitch jump than moving toward the sound source. For example, for car at speed of 120 km/h (33.3 m/s) frequency is increased at ratio of 1.097 (160 cents) when moving towards the sound source; it is decreased at ratio of 1.11 (−177 cents) when moving away from the source. For very high v_{obs} this difference is more remarkable. If $v_{obs} = 0.5c$ (171.5 m/s or 617.4 km/h) the upward pitch jump is by musical fifth, and downward pitch shift by musical octave.

For the case of source moving with velocity v_{sc} the actual (real) decrease in wavelength in direction of source movement is like the sound speed c was decreased by the velocity v_{sc}:

$$\lambda = (c - v_{sc})/f. \tag{2.48}$$

However, the speed of propagation of this wave is c, therefore all observers will perceive frequency $f = c/\lambda$.·Frequency f_{gen} generated that way and received by all observers will be:

$$f_{gen} = c/(c - v_{sc})\cdot f_{sc}. \tag{2.49}$$

Similarly for sound wave behind the source propagated in the direction opposite to the direction of movement:

$$f_{gen} = c/(c + v_{sc})\cdot f_{sc}. \tag{2.50}$$

If both observer and sound source move towards each other, the superposition gives change in frequency as:

$$f_{obs} = ((c + v_{obs})/c)\cdot f_{gen} = (c + v_{obs})/(c - v_{sc})\cdot f_{sc} \tag{2.46}$$

The third factor which can be taken into account in Doppler effect is the constant move flow of the medium. It can be analyzed formally in similar way to analysis of the movement of the observer.

2.12. Wave interference

Interference is a phenomenon of influencing and disturbance of a sound wave by identical waveform which reached the same area in space. An example of interference is given in Fig. 2.35

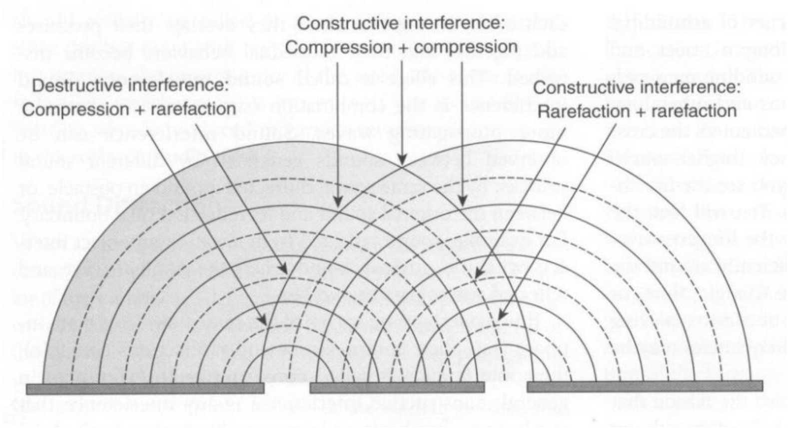


Fig. 2.35. Sound interference of two identical spherical. Solid lines indicate plases of compression in wave, and dashed lines places of rarefaction. Coincidence of compression areas leads to increased compression and similarly coincidence of rarefied areas leads to the increased rarefaction. In places where compression and rarefaction phases overlap the wave cancels each other. From reference [1].

Interference will always take place when identical soundwave patterns will overlap. This happens when two sound sources at different space locations produce identical soundwaves. While it is difficult to make such a sound sources by technical means in many real circumstances conditions for interference occur because of reflections and diffraction. Reflection creates sound source image of correlated waveform pattern with some amplitude scaling (due to attenuation at the reflection). This makes conditions for interference. Very straight coherent sources of spherical wave are shown in Fig. 2.35. These coherent two sources producing strong interference pattern result from diffraction of single plane or spherical wave on narrow slits. Coincidence of compression (solid lines) produced by one source with compression produced by the other source leads to increased compression in places where solid lines cross. Coincidence of rarefaction produced by one source with rarefaction produced by the other source, leads to increased rarefaction (crossing dashed lines). These are places denoted as 'constructive interference' in Fig. 2.35. Coincidence of rarefaction and compression (crossings of dashed and solid lines) cancels each other. These are places denoted as 'destructive interference' in Fig. 2.35.

Interference is always possible when overlapping sound waves are coherent what means that there is strong correlation between them. In the sound field this is usual a sound pattern and its image coming from a reflection or diffraction. Adding such waveforms makes amplitude doubled in all places of constructive interference (+ 6 dB), and amplitude is zeroed in places of destructive interference because of compressions and rarefactions that cancel each other. When two waver are uncorrelated then the addition is purely energetic (+3 dB).

2.13. Standing waves

2.13.1. Reflection from one wall

Creation of standing waves is the most important effect of the interference. Standing wave is an important issue in room acoustics and is treated in details in the section on concert halls and room acoustics. Chances for standing wave creation are always when there is a reflection for a wall. The boundary conditions impose that there is no particle displacement at the wall (as wall impedance $z = \infty$). A reflected wave having the same frequency and traveling in the opposite direction is superimposed on initial sound wave:

$$p_1(x,t)+p_2(x,t)=A_1\sin(2\pi ft - kx) + A_2\sin(2\pi ft + kx),\ \mathrm{k} = 2\pi/\lambda \tag{2.47}$$

Assuming for simplicity that $A_1 = A_2 = A$:

$$p_1(x,t)+p_2(x,t)=2A\cos(kx)\cdot\sin(2\pi ft) \tag{2.48}$$

Nodes of standing wave where the pressure vanishes occur at

$$x_n = (2n\text{-}1)\lambda/4\ (n = 1,2,3,\ ..). \tag{2.49}$$

These are the points in space (it was assumed that the wall is at location $x = 0$) where function $\cos(kx) = 0$. Maximum sound pressure of standing wave occurs at locations:

$$x_n = n\lambda/2,\ (n = 1,2,3,\ ..). \tag{2.50}$$

The possible excitation of standing waves indicates that sound pressure measurements close to the wall can be inaccurate as the standing wave may cause an increase in the amplitude by up to +6 dB. Effective excitation of standing wave depends on the position of sound source: for pressure source − points at locations as in Eq. (2.50); for velocity source (oscillating) source − points at locations as in Eq. (2.49).

2.13.2. Reflections from two walls

Different standing wave occurs between two parallel walls facing each other at distances 0 and L. Boundary conditions at two walls impose that a standing wave can be created for all frequencies related to wavelengths $\lambda = 2L/n$ (n = 1, 2, 3, …). Set of resonant frequencies corresponding to series of standing waves between two walls separated by L is harmonically related:

$$f_\mathrm{n} = c/\lambda = (c/2L)\cdot\mathrm{n}. \tag{2.50}$$

Such a standing wave is an unwanted phenomenon in room acoustics especially affecting sound in so called small rooms.

3. Sound sources

Radiation from sound sources is an important section in acoustical analysis involving considerable mathematics. In theoretical considerations, all sound sources can be represented by closed surface surrounding the source having at all points predefined sound pressure and particle velocity in such way that the external sound field is identical to that produced by analyzed sound source. Natural vibrating bodies which often are the sound sources, have complex modes of vibration. These sound sources can be very difficult to analyze mathematically. In many instances, simplification to sources of simpler modes of vibration is sufficient approximation. Therefore, analysis of such mathematically simple sources as pulsating sphere, oscillating sphere and vibrating rigid piston are considered to be classical problems in acoustics. Second class of sources are monopole sources, linearly positioned monopole sources, dipoles and quadrupoles. In many instances these highly theoretical sound sources are items for simulations of real sound sources or sources developed by engineers. For instance, a monopole or pulsating sphere well represent small sources in low frequency range, oscillating sphere represents noise sources created by vibrating parts of machinery, a vibrating piston is good representation of such products like the loudspeaker.

3.1. Pulsating sphere and monopole point source

Pulsating sphere and monopole point source are the simplest acoustical sources propagating the spherical wave. Pulsating sphere of radius *a* is shown in Fig. 3.36.

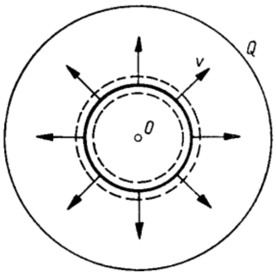


Fig. 3.36. Pulsating sphere of radius *a*, and velocity *v* at the surface. Q is the directional pattern of the source. From reference [8].

The monopole point source is the theoretical idealization of pulsating sphere by setting $a \rightarrow 0$. Such idealization is convenient as for a sphere of radius *a* the acoustical field in not defined for $r < a$. The point source is the easiest sound source to model theoretically but it is the most difficult source to construct. In laboratory equipment, the pulsating sphere is approximated by a twelve-speaker spherical sound sources such as one shown in Fig. 3.37.

Fig. 3.37. Twelve-speaker omnidirectional sound source: a laboratory construction of pulsating sphere. Bruel & Kjaer Omni Power Sound Source - type 4292.

The pulsating sphere and monopole point source are described by the following sound pressure and particle velocity (in case of the sphere these quantities refer to the surface of the sphere). For monopole generated sound pressure and particle velocity are:

$$p = j\omega\rho_0 \frac{Q}{4\pi r} e^{j(\omega t - kr)} \tag{2.51}$$

$$v = (1 + jkr)\frac{Q}{4\pi r^2}e^{j(\omega t - kr)}$$

(2.52)

In these equations the Q is a new element. The Q is called point source strength. For the sphere of a radius a these equations hold with source strength

$$Q = \frac{4\pi a^2 v}{1 + jka}e^{jka}$$

where v is the velocity of the sphere surface.

3.1.1. Pulsating sphere radiation impedance

Most important quantity characterizing the sound source is its radiation impedance z_r. For pulsating sphere

$$z_r = \rho_0 c\left(\frac{(ka)^2}{1 + (ka)^2} + j\frac{ka}{1 + (ka)^2}\right).$$

(2.52)

Radiation impedance describes the sound propagation from the source, in this case the spherical sound source. Assuming the sound source is driven by some electrical circuit, the radiation impedance is that impedance, which is seen by the circuit, for instance power amplifier as a terminating load to the circuit. Radiation impedance is usually graphically presented in a form of reduced radiation resistance and reactance Z = $R_j + jX_j = z_r/\rho_0 c$. This is to unify the functions for all media of propagation (gases and liquids).

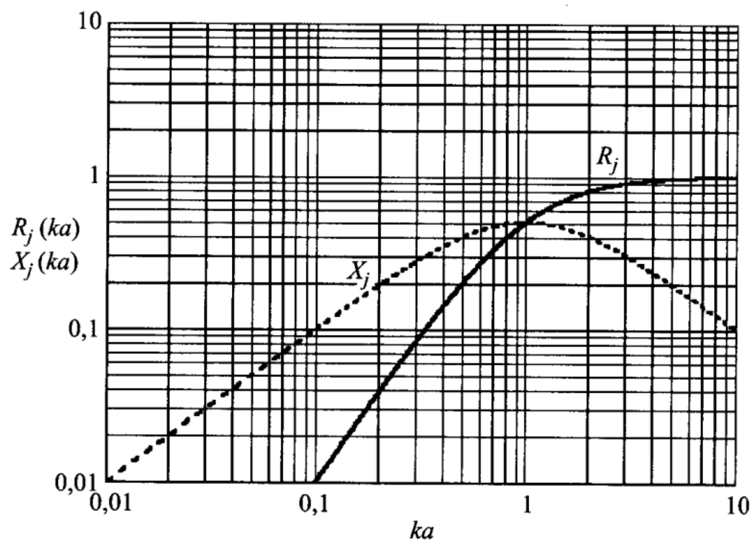


Fig. 3.38. Radiation resistance and reactance of pulsating sphere of radius a. From reference [9].

Radiation resistance represents ability of the source to propagate sound into the sound field. Radiation reactance represents the energy transferred to the velocity of the particles in the direct vicinity of the source; however, this vibration is not transferred farther to the field. Rather energy is returned to the sound source. Functions in Fig. 3.38 show that pulsating sphere reaches its full effectiveness in propagation of energy to the sound field for $ka > 3$. This corresponds to the radius of the sphere equal or larger than 0.5λ. For $ka = 1$ (a = 0.16λ), there is the maximum value of reactance, thus highest interference with the nearest layers of medium but this energy is not well transferred to the medium in the far filed. At this point the resistance equals 0.5, thus 50% of its maximum value. Below a = 0.16λ (ka = 1), the reactance is larger than resistance showing that there is more energy in near field than transferred to the far field. In practice, to record any signal in such a low frequency range it is necessary to listen to or place the microphone close to the surface of the source. To give same imagination on numbers: a spherical sound source of 5.5-cm radius (diameter of an 11-cm ball) in air is totally ineffective in generating 100-Hz wave (R_j = 0.01), at 1 kHz R_j = 0.5, and interaction with neighboring space is the highest. Finally, for 3 kHz and above the source reaches its maximum propagation to the medium.

For the spherical sound source and ka < 0.3 (low frequency range), domination of reactance means that almost all energy is merely transferred to the mass of air just around the source. This mass reactance is estimated as $M_r = 4\pi a^3 \rho_0$. Thus, it is three times larger than the mass of air enclosed inside the spherical source ($4\pi a^3 \rho_0/3$). Mass reactance is negligible in high frequency range.

Monopole point source in an important theoretical model of the spherical wave sound source since combination of such sources is used to model sound sources of more complex directional properties. These models include dipoles, quadruples or linearly arranged sources of various kind and mutual phases of pulsation.

3.2. Oscillating spherical source and dipole source

Oscillating sphere is the second sound source out of set of basic sound sources treated analytically. This source is easy to be constructed, and actually many elements in machinery which are sources of noise can be modelled as oscilating shperical source. Oscillating sphere of radius *a* is shown in Fig. 3.39.

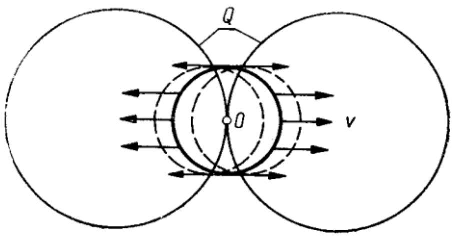


Fig. 3.39. Oscillating sphere of radius *a*, and velocity *v* of oscillation. Q is the directional pattern which is bidirectional. From reference [8].

The dipole consisting of two monopole point sources acting in opposite phases located at infinitively small distance from each other is the theoretical idealization of oscillating sphere sound source (Fig. 3.40). This construction shows that the dipole sound source and oscillating sphere sound source have bidirectional directional pattern governed by $\cos\Theta$ function. A comparison between the pulsating sphere and oscillating sphere directional pattern in dB scale is shown in Fig. 3.41.

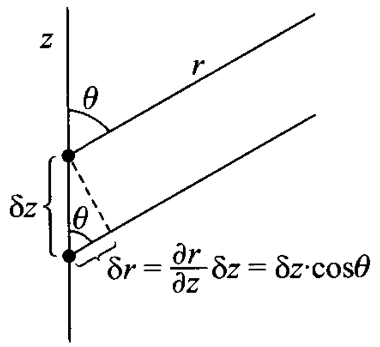


Fig. 3.40. Geometrical construction of monopole sound sources acting in opposite phases to represent infinitively small oscillating sphere. From reference [9].

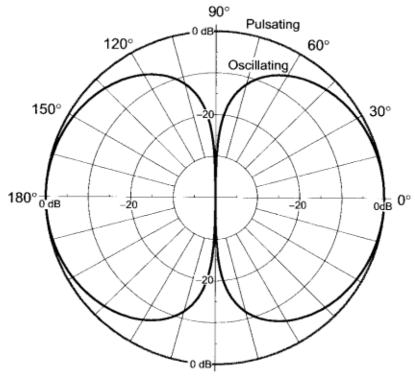


Fig. 3.41. Comparison of directional patterns for pulsation and oscillating spherical sound sources. From reference [10].

Equations for the sound pressure and particle velocity of the oscillating sphere and dipole source are more complex and can be found in suggested readings.

3.2.1. Oscillating sphere radiation impedance

Radiation impedance of oscillating sphere is given by

$$z_r = \frac{1}{3}\rho_0 c\left(\frac{(ka)^4}{4+(ka)^4} + j\frac{ka(2+(ka)^2)}{4+(ka)^4}\right). \tag{2.53}$$

Reduced radiation resistance and reactance Z = $R_j + jX_j = z_r/\rho_0 c$ is shown in Fig. 3.42. There are several differences in wave propagation by oscillating (Fig. 3.42) and pulsating sphere (Fig. 3.38) worth noting.

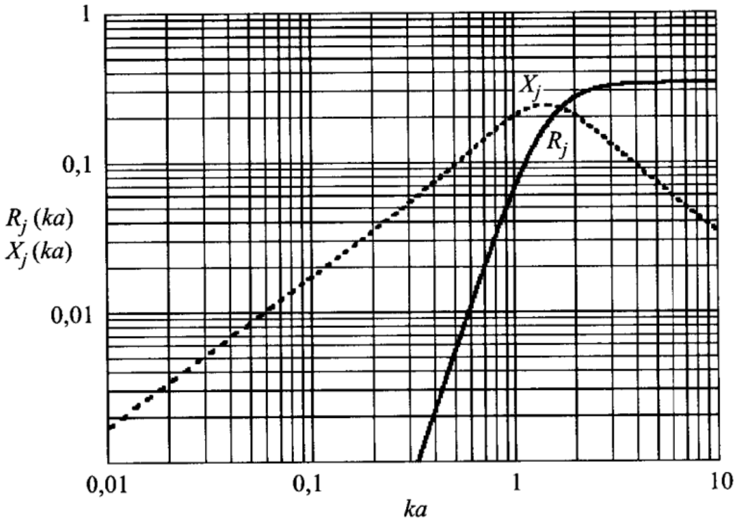


Fig. 3.42. Radiation resistance and reactance of oscillating sphere of radius a. From reference [9].

Even for infinitely small λ/a (large ka) radiation resistance of oscillating sphere does not reach $R_j = 1$.

Its full effectiveness in propagating energy to the sound field for $ka > 3$ is 1/3, three times poorer than for pulsating sphere. As can be noticed from Figs. 3.39 and 3.40 this is because only for $\Theta = 90^o$ and 270^o $\cos\Theta = 1$, angles which point along the main axis of oscillation. At these angles the sphere surface is perpendicular (normal) to direction of movement. For $\Theta = 0^o$ and 180^o there is a grazing angle od sphere surface with direction of oscillation and then no wave is generated. The eight-like shape of directional pattern results in less effective overall transfer of energy to the far field. There is also noticeable difference in R_j as compared to pulsating sphere for $ka < 2$. The R_j diminishes with fourth power of *ka*. Thus the effectiveness of the source for large wavelengths is much smaller than in case of pulsating sphere. In contrast, the temporary transfer of energy to nearfield is larger for the oscillating sphere as the X_j is twice as large as for pulsating sphere in low frequency range (small *ka*).

3.3. Oscillating piston in an infinite baffle

Mathematical analysis of an oscillating piston as a sound source can be done in several conditions of vibration. One condition is to analyze piston which oscillates freely (alone). In this condition, efficiency of propagation in low frequecy range is small and the source is somewhat similar to oscillating sphere. It is more proper and common to analyze piston oscillation when it is fit to an infinite rigid not oscillating baffle. In such condition a piston of radius *a* oscillates at velocity *v* but the baffle rigidly stays at $x = 0$ (baffle velocity $v_b = 0$). The hafspace in the front of the piston is separated from the half space at the back what allow for efficient radiation in low frequency range. It is a standard for loudspeaker systems to work in boxes separating front of the loudspeaker from its back to enhance radiation in for frequencies. Oscillating piston in an infinite baffle is thus correct theoretical representation of the loudspeaker in its normal conditions of work. Third condition is to consider the piston oscillating at the end of the long pipe. This is to model real situation of many kinds, for instance action of wind musical instrument or organ pipe where radiation from the pipe end can be treated as radiation from a piston attached to the pipe. Another practical application is the output from the ventilation duct which is usually a source of noise. Actually an end correction for Helmholtz rezonator discussed in section 1.6 is also derived from the radiation of a piston. The air movement in the resonator neck is treated as a vibration of a piston.

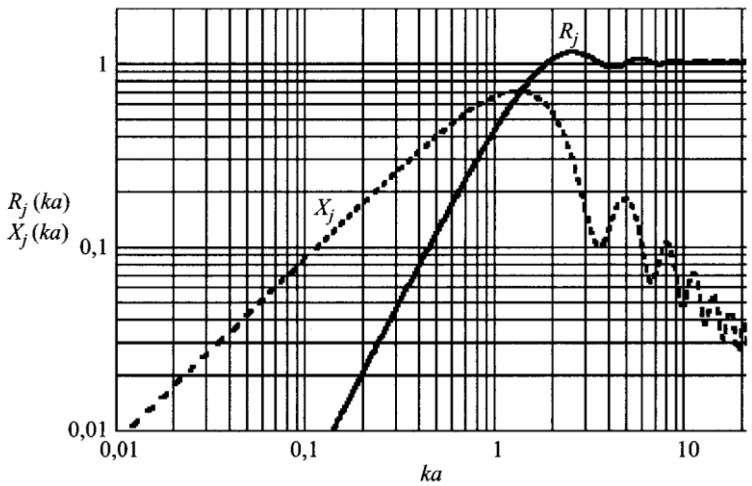


Fig. 3.43. Resistance and reactance of the oscillating piston of radius *a* in an infinite baffle. From reference [9].

In the following, some examplesof solutions will be given for a condition of piston vibrating in the infinite baffle. It is assumed that the piston is rigid (oscillates equally across its entire surface), and that the baffle is still. To get pressure or velocity at any given point in space it is necessary integrate partial pressures coming from the subsequent points on a piston. These leads to specific algebraic solutions which are complex enough to refer the reader to numerous other literature.

3.3.1. Oscillating piston radiation impedance

The radiation resitance and reactance of a piston in rigid baffle are shown in Fig. 3.43. Qualitatelively the functions are quite similar to R_j, X_j for the pulsating sphere. The resistance R_j slope is steeper than that for the sphere and lets the function reach value of 1 at smaller *ka* values than for the pulsating sphere. The piston in a baffle more effectively radiates sound for $1 < ka < 3$ than pulsating sphere. A comparison of a piston to oscillating sphere reveals that in high frequency range ($ka > 3$) radiation resistance is about 1, the three times larger value than for oscillating sphere. It is like for the oscillatoing sphere. This effect is a result of applying infinite baffle around the oscillating piston.

3.3.2. Directivity pattern

For the piston, the directivity pattern in the far field (further away than about 10 wavelengths λ from the center of the piston) is well defined (i.e. is remains the same at any value of radius *r*) and different at low and high frequencies. Change in directivity pattern with frequency (i.e. with wavelength λ compared to piston radius *a*) makes oscillating piston different from the pulsating and oscillating sphere discussed earlier. Due to interference of waves incoming from various points on the piston surface to the point in space at distance *r* from the piston the directivity pattern (Fig. 3.44) alters from omnidirectional for $ka < 1$ ($\lambda > 6a$) to highly directional for $ka > 4$ ($\lambda < 1.5a$) with several side lobes on both sides of the main lobe.

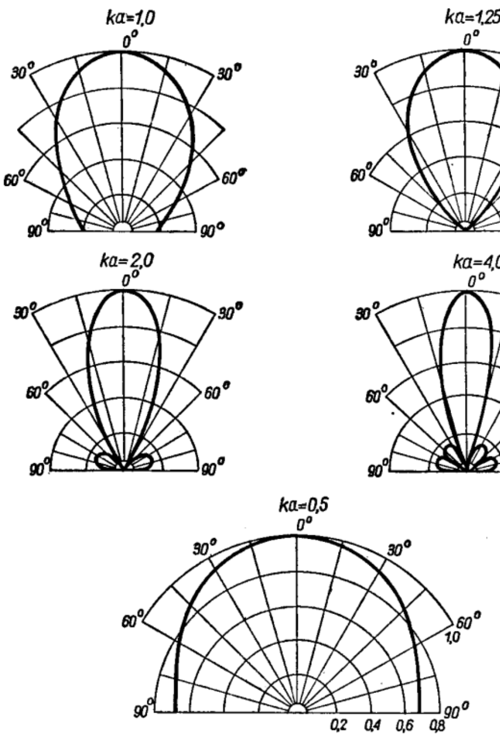


Fig. 3.44. Directional patterns for the oscillating piston of radius *a* in an infinite baffle for various *ka* values. From reference [8].

In the near field, the sound field is very complex because of the interference of waves produced by different points on the piston surface. In the literature, exact calculation of pressure change with distance exists only for points on axis of piston rotational symmetry (normal line in the piston center).

Example of results of such calculations is shown in Fig. 3.45 for a piston of a radius $a = 4\lambda$ (piston diameter $2a = 8\lambda$). This is fairly wide piston as it corresponds to piston of 27-cm diameter generating 10-kHz signal (λ = 3.4 cm, compare the loudspeaker sizes used for 10 kHz signal). Nevertheless, the changes seen in the pressure along the normal distance from the center of the piston are substantial. In the nearfield, for distance smaller than the diameter $2a$ pressure P exhibits strong variations between 0 and the maximum value. This variation comes from the strong interference of waves at arriving at different phases from various points on the piston. This phase variation is not that large at long distances from the piston i.e. in the far field. Dashed line representing amplitude approximation valid in the far field shows this. At distances $r > 30\lambda$ (at about 1 meter for the example with 10 kHz signal) amplitude change shown by dashed line decreases smoothly with r. This approximation is not correct in the near field.

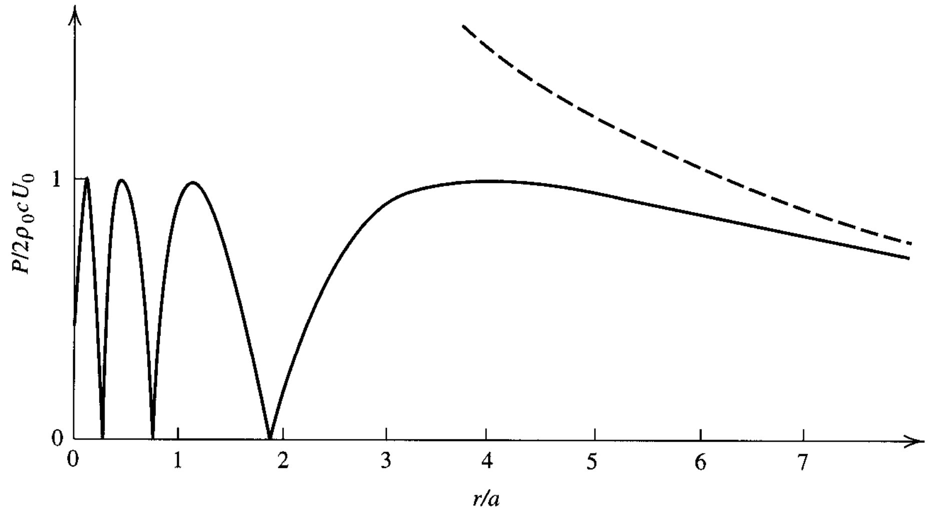


Fig. 3.45. Variation of pressure amplitude (solid line) on the axis of symmetry of vibrating piston of radius $a = 4\lambda$ (diameter of 8λ). In the nearfield, at a distance from the piston smaller than the diameter $2a = 8\lambda$ pressure P exhibits strong variations between 0 and the maximum value. Dashed line represents amplitude approximation valid in far field, for $r > 30\lambda$. From reference [11].

Theoretical consideration referring to the oscillating piston are considered important as this simple model source is quite good approximation for many practically existing sound sources.

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